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Generalization of Feedback Error Learning to MIMO Systems and its Stability Proof

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This work is dedicated

to my parents: Abdrabalameer and Jamilah,

to my brothers: Bassam and Basem,

to my sisters: Samaher, Assel, Basmah and Ayat,

and to my grandparents.

Generalization of Feedback Error Learning to MIMO Systems and its Stability Proof

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Abstract

Learning control is a promising control method since it provides a good performance by on-line learning, without a mathematical model of the object to be controlled. Recently, a new learning control scheme named feedback error learning (FEL) has been proposed and attracted much attention. A striking feature of this scheme is that it uses a feedforward controller which is adjusted by some learning law. This thesis generalizes the FEL scheme to multi-input multi-output (MIMO) systems and then proves the stability of the generalized scheme. The concept of interactor matrix plays a key role to treat MIMO systems not necessarily biproper.

Firstly, the existence of an ideal inverse of the system is shown by means of the interactor. Since the system is not necessarily invertible with properness, the inverse is attained by a feedforward controller with compensation by a pre-filter. Secondly, the learning law for the feedforward controller is given so that the parameter tends to the ideal value. Thirdly, the thesis gives two kinds of stability proof in order to guarantee convergence of the parameter. Numerical simulation for various examples has been performed to show the effectiveness of the scheme.

Keywords: Learning Control, Feedback Error Learning, Adaptive Control, MIMO Systems, Stability Analysis.

フィードバック誤差学習の多入出力系への一般化と安定性の証明

バセル　アルアリ

内容梗概

学習制御は、制御対象の数理モデルを用いることなくオンライン学習によって良い性能を与えるため、有望な制御手法である。最近、フィードバック誤差学習（FEL)と呼ばれる新しい学習制御手法が提案され、注目を浴びている。この手法の著しい特長は、フィードフォワード制御器を学習に用いている点にある。本論文では FEL を多入出力系に拡張し、さらにその安定性を証明する。必ずしもバイプロパでない多入出力系を扱うためにインタラクタ行列の概念が重要な役割を果たす。

最初にインタラクタを用いて理想的な逆システムの存在が示される。逆システムは必ずしもプロパではないので、これはフィードフォワード制御器とプレフィルタの補償によって達成される。次に、パラメータがこの理想的な値に近づくようにフィードフォワード制御器の学習則を与える。さらに、このパラメータの収束性を保証するために安定性を 2 種類の方法で示す。様々な例に対して数値シミュレーションを行い、本手法の有効性が示された。

キーワード： 学習制御，フィードバック誤差制御．適応制御，
多入出力システム，安定解析

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Chapter 1
INTRODUCTION

1.1 Background

The main objective of a control system is to make the plant output y behave in a desired way by manipulating the plant input u. There are mainly two control problems. First, the regulator problem is to manipulate u to counteract the effect of a disturbance d. Secondly, the servo problem is to manipulate u to keep the output close to a given reference input r. Thus, in both cases we want the control error $e = y - r$ to be as small as possible in order to have good performance. The design of a controller is generally based on a model of the plant. Therefore, a more accurate model of the plant is required in order to improve the controller. However, due to factors like process uncertainties, process non-linearity or time varying parameters, the identification and modelling that are needed might be difficult, expensive or sometimes even impossible. When modelling a plant, the following problems can be encountered [8]:

- The system is too complex to understand or to represent in a simple way.
- The model is difficult or expensive to evaluate. The characteristics of some (non-linear) effects may be hard to obtain.
- The plant may be subject to large environmental disturbances, which are difficult to predict.
- The plant parameters may be time varying.

Robust control can offer a good solution for this kind of problems when the structure of the plant dynamics and the disturbances are known, but the values of some parameters can not be determined. The design process of robust control is normally assumed that a complete knowledge of the plant is available so that the feedback controller is chosen to maintain the closed loop stability and obtain good performance. However, the structure of the plant dynamics in most of the applications is not known, namely when we have only rough information about the plant or many parameters can not be determined. The feedback controller can maintain the closed-loop stability but it is difficult to track the desired performance if there is no enough knowledge about the plant. Thus, the learning control can be considered in order to improve the performance of the system by accomplishing the required knowledge.

In learning control, the controller is not designed on the basis of the process model. The controller is either trained based on the previously collected data or is trained during control. The former can be considered as an off-line learning while the latter is an on-line learning. In both cases, the controller is implemented as a feedforward controller. Therefore, it should be noted that the main objective of the feeforward controller is to improve the performance of the system while the feedback controller is to maintain the closed-loop stability.

The origin of learning feedforward control can be traced back to a scheme known as Feedback Error Learning (FEL), as illustrated in Fig.1. This scheme was proposed by Kawato et al. [1]. The main objective of FEL is to

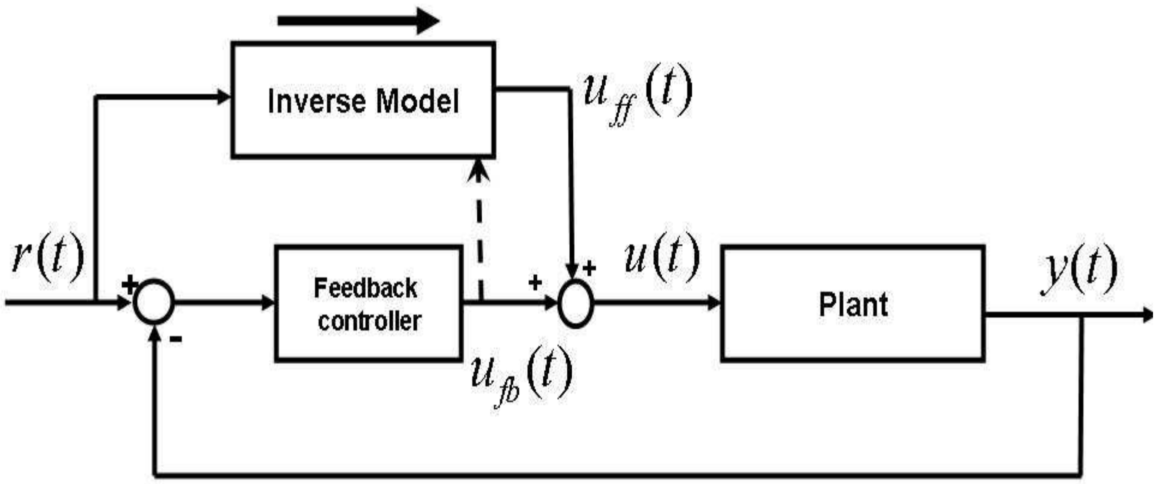


Fig. 1.1 Feedback Error Learning Scheme (from Kawato, 1990 [5])

implement the feedforward controller as a function approximator, instead of designing linear control on the basis of a plant model. The types of function approximator which have been implemented in [1], [6] as an example have been Multi Layer Perception (MLP) and Cerebellar Model Articulation Controller (CMAC) neural networks, respectively. The main result in [1] is that the inverse dynamic model is acquired by repetitively experiencing a single motor pattern, while receiving the desired trajectory and monitoring the feedback torque as an error signal. Also, it is mentioned that as motor learning proceeded, the inverse dynamics model gradually takes the place of the external feedback as a main controller. The second result is interesting because it is logical that once we achieved the inverse of the system, the feedback controller or the error signal will disappear.

Furthermore, Kawato in [5] trains a neural network by using the error signal and this approach was applied successfully in robotics application. It is also shown mathematically that feedback error learning approach can be considered as a Newton-like method in functional space.

However, the practical value of a learning controller depends on what type of function approximator is used. Several problems exist for this kind of function approximator (MLP or CMAC) such as slow convergence, local minima, computational load and high memory requirement [7].

On the other hand, applications have shown that the FEL controller gives a considerable improvement on the performance of the system [1], [5], [6], [11]. Moreover, FEL can obtain a high tracking performance without extensive modeling. The novelty of FEL lies in its use of feedback error as a teaching signal, which is essentially new in control literature.

Recently, Miyamura and Kimura [2] have established a control theoretical validity of the FEL method in the frame of adaptive control, proving its stability based on strictly positive realness, whereas Muramatsu and Watanabe [3] have relaxed the positive realness condition of FEL by using the error signal between the reference and the output signal as well as the feedback input. They have used an adaptive linear filter for function approximator, and did not use MLP nor neural networks any more, but still the learning of the inverse model has been effectively used in [2], [3].

1.2 Research Goals

In view of the fact that most of the process control applications are multi-input multi-output (MIMO), we generalize FEL for a MIMO system using simple linear system parameterization of the feedforward control as a function approximator.

The objectives of the present research are three-folds. At first, an ideal inverse of the system is derived using the concept of interactor (Wolovich and Falb [15]). This concept was originally introduced to generalize the relative degree of scalar transfer functions to MIMO systems.

Secondly, the learning law for feedforward controller based on generalization of FEL for MIMO system is derived using the ideal and adaptive feedforward controller. The parameter of the learning law has a matrix form instead of a vector form.

Thirdly, the stability of the derived learning law has been proved with and without the positive realness condition. In both cases, we obtain the same conclusion that the parameter of the learning law converges to the ideal value. As a result, asymptotic relation between the ideal feedforward based on the interactorization concept and adaptive one based on FEL is achieved.

1.3 Contributions

The main contribution of this research can be summarized as follows:

1. Derive an ideal feedforward controller using inverse interactorization concept.

2. Derive an adaptive feedforward controller using linear parameterization and obtain its tuning rule by generalizing feedback error learning to MIMO systems.

3. Prove the stability of the derived learning law with and without positive realness condition.

1.4 Document Overview

In Chapter II, we give an overview about MIMO system, state the problem and derive the ideal feedforward for the MIMO system utilizing the interactor concept. In Chapter III, we study the main feedforward learning control techniques which are available in the literatures. We make comparison between these techniques and emphasize more about the analysis and the stability of feedback error learning. We generalize FEL for MIMO system by deriving an adaptive feedforward and obtain its learning law in Chapter IV. In Chapter V, we investigate the stability of the derived MIMO-FEL learning law with and without positive realness condition. In Section VI, simulation results are shown to evaluate the effectiveness of the proposed method. Section VII analyzes the results and presents the main achievements of this research.

Chapter 2
Problem Formulation

2.1 Introduction to MIMO system

We consider a multi-input multi-output (MIMO) plant with m-inputs and m-outputs (i.e., square system). Thus, the transfer function model is $y(s) = G(s)u(s)$, where $y \in R^m$, $u \in R^m$, *and* $G(s) \in R^{m \times m}(s)$ is a transfer function matrix as shown in Fig. 2.1.

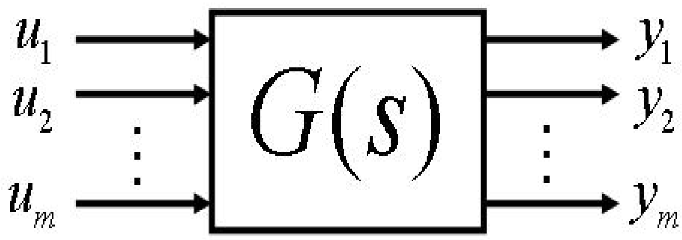


Fig. 2.1 MIMO system

If we change one input, say u_1, then this will generally affect all the outputs $y_1, y_2, \cdots, y_m$. Namely, there is an interaction between the inputs and the output which is very common in a real situation. On the other hand, a non-interacting plant would result if u_1 affects only y_1, u_2 affects only y_2, and so on, which rarely occurs in the real situation.

A main difference between a scalar (SISO) and a MIMO system is the presence of the direction in the latter. Directions are significant for vectors and matrices, but not for scalars. By solving the directions problem, most of the ideas and techniques may be extended to MIMO systems.

There are three useful rules when evaluating transfer functions for MIMO systems as mentioned in [25] which can be seen in App. A.

To illustrate the idea of MIMO rule, we will consider FEL scheme in Fig. 1.1 while the plant $P(s)$, the inverse model $Q(s)$ and the feedback gain K_{fb} are considered to be MIMO system with appropriate dimensions. Therefore, the overall closed-loop transfer matrix from r to y is obtained as follows:

$$y(t) = P(s)[Q(s)r(t) + K_{fb}(r(t) - y(t))]$$
$$y(t) = [I + P(s)K_{fb}]^{-1}[P(s)Q(s) + P(s)K_{fb}]r(t).$$
$$G(s) = [I + P(s)K_{fb}]^{-1}[P(s)Q(s) + P(s)K_{fb}]. \tag{2.1}$$

Detailed information can be seen in Chapter 4.

2.2 Statement of the Problem

It can be seen clearly from the previous section that if $Q(s) = P^{-1}(s)$, then we will have perfect tracking. Therefore, the question is now how we can find the inverse of the plant if the plant is a MIMO system. Furthermore, what will happen if the plant is not invertible with properness, or if the transfer matrix is strictly proper.

At first, we will find the ideal inverse of the system by means of interactor concept [14]. The main idea of ideal inverse of the system is assuming that we have good representation of system or accurate approximation of the system parameters by means of system identification and modelling. So, based on this approximated parameters, we will derive the inverse of the system by including a pre-filter in order to compensate the properness of the system. This pre-filter will equal to the identity matrix if the transfer matrix is biproper. Otherwise, it will be equal to the inverse of the interactor.

The main idea can be explained mathematically as follows:

$$P(s)Q(s) = W(s). \tag{2.2}$$

Given $P(s)$, we want to make $W(s)$ as "close" to the identity as possible by choosing (bi)proper $Q(s)$. If $P(s)$ is invertible, then $W(s)$ will be identity matrix. However, if $P(s)$ is not invertible, how can we find the $Q(s)$ such that $W(s)$ will be diagonal or inverse of the interactor. The whole scenario will be derived in the next section.

2.3 Ideal MIMO Feedforward Control

Consider an n-dimensional controllable and observable linear system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t), \tag{2.3}$$

with m-inputs and m-outputs (i.e., square system). Assume that the system is of minimal phase, and hence it has no invariant zeros at the origin, namely

$$\det\Gamma \neq 0, \qquad \Gamma := \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{2.4}$$

Furthermore, if $\det D \neq 0$, then the system is invertible with properness and hence generalization of FEL is straightforward, so that our method below significantly simplified. On the other hand, if $\det D = 0$, then the generalization is rather difficult, since we need to define an appropriate pre-filter. We will overcome the difficulty below. In order to explain our method clearly, we will further assume $D = 0$ in what follows. (If $\det D = 0$ and $D \neq 0$, then the argument will be more complicated but our basic idea still holds.)

The transfer matrix of the system (2.3) is $P(s) := C(sI - A)^{-1}B$. Also, we assume that

$$\Lambda := \begin{pmatrix} c_1 A^{\mu_1 - 1} B \\ \vdots \\ c_m A^{\mu_m - 1} B \end{pmatrix} \tag{2.5}$$

is invertible, where c_k is the *k-th* row vector of C, and μ_k is the minimal integer such that

$$c_k A^{\mu_k - 1} B \neq 0, \quad \mu_k \geq 1, \quad \text{for} \quad k = 1, \cdots, m. \tag{2.6}$$

The integers $\mu_1, \mu_2, \cdots, \mu_m$ can be regarded as a generalization of the relative degree. We assume that these integers are known. Furthermore, they have a close relationship to the following concept (Wolovich and Falb [14]).

Definition 1. Given $P(s)$, a square polynomial matrix $L(s)$ is called an interactor if

$$\lim_{s \to \infty} L(s)P(s) \tag{2.7}$$

is a finite matrix with full rank.

If Λ in (2.5) is invertible, we can set the interactor matrix to a diagonal form

$$L(s) = \begin{pmatrix} (s + a_1)^{\mu_1} & & 0 \\ & \ddots & \\ 0 & & (s + a_m)^{\mu_m} \end{pmatrix}, \tag{2.8}$$

where μ_k is as defined before, and a_k is an arbitrarily chosen positive real number for $k = 1, \cdots, m.$

We find the inverse of the system using the inversed interactorization concept. For the original system (2.3) and the interactor $L(s)$ in (2.8) with arbitrarily chosen $a_k (k = 1, \cdots, m)$, there exists a dummy gain F such that the following equality holds

$$N(s) = C(sI - A + BF)^{-1} B = L(s)^{-1} \Lambda . \tag{2.9}$$

The dummy gain F can be obtained by the simple formula (Mutoh [14]):

$$F := \Lambda^{-1} [L_0 \cdots L_\mu] \begin{bmatrix} C \\ CA \\ \vdots \\ CA^\mu \end{bmatrix}, \tag{2.10}$$

where

$$L(s) = L_0 + L_1 s + \cdots + L_\mu s^\mu , \tag{2.11}$$
$$\mu := \max(\mu_1, \cdots, \mu_m) . \tag{2.12}$$

Furthermore, we consider that the pre-filter will be equal to the inverse of the interactor such that the following equality holds

$$W(s) = L(s)^{-1} = \begin{pmatrix} \frac{1}{(s+a_1)^{\mu_1}} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{(s+a_m)^{\mu_m}} \end{pmatrix} = N(s)\Lambda^{-1}. \tag{2.13}$$

As a result, if we take the feedforward controller $Q(s)$ equal to

$$Q(s) = \left[I - F(sI - A + BF)^{-1} B \right] \Lambda^{-1}, \tag{2.14}$$

then we have

$$P(s)Q(s) = W(s) , \tag{2.15}$$

which can be easily verified. Therefore, the ideal feedforward control is:

$$u_0(t) = Q(s) r(t) . \tag{2.16}$$

Based on the above, we derive the following lemma:

Lemma 1:
If we define the feedforward controller $Q(s)$ equal to
$Q(s) = \left[I - F(sI - A + BF)^{-1} B \right] \Lambda^{-1}$,
then we have
$P(s)Q(s) = W(s)$.

Proof of Lemma 1:
We need to check that the following equality holds in order to prove the above lemma:

$$P(s)Q(s) = W(s)$$
$$C(sI - A)^{-1} B \left[I - F(sI - A + BF)^{-1} B \right] \Lambda^{-1} = C(sI - A + BF)^{-1} B \Lambda^{-1}$$

Therefore, we have

$$C(sI - A)^{-1} B \left[I - F(sI - A + BF)^{-1} B \right] \Lambda^{-1} - C(sI - A + BF)^{-1} B \Lambda^{-1} = 0$$

Taking the LHS

$$\begin{aligned}
LHS &= C(sI - A)^{-1} B \left[I - F(sI - A + BF)^{-1} B \right] \Lambda^{-1} - C(sI - A + BF)^{-1} B \Lambda^{-1} \\
&= C\left\{ (sI - A)^{-1} B - (sI - A)^{-1} BF(sI - A + BF)^{-1} B - (sI - A + BF)^{-1} B \right\} \Lambda^{-1} \\
&= C\left\{ (sI - A)^{-1} B - [(sI - A)^{-1} BF + I](sI - A + BF)^{-1} B \right\} \Lambda^{-1} \\
&= C\left\{ (sI - A)^{-1} B - [(sI - A)^{-1} (BF + sI - A)](sI - A + BF)^{-1} B \right\} \Lambda^{-1} \\
&= C\left\{ (sI - A)^{-1} B - (sI - A)^{-1} B \right\} \Lambda^{-1} = 0 = RHS .
\end{aligned}$$ □

In order to clarify the above procedure of the ideal feedforward, we consider the following example.

Example 1:
Giving

$$A = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -4 & 1 \\ 3 & 1 & -5 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix},$$

we have the following transfer matrix of the plant:

$$P(s)=\begin{pmatrix} \dfrac{s^2+4s+1}{s^3+8s^2+19s+23} & \dfrac{-2s-19}{s^3+8s^2+19s+23} \\ \dfrac{2s^2+9s+7}{s^3+8s^2+19s+23} & \dfrac{s^2+5s-18}{s^3+8s^2+19s+23} \end{pmatrix}.$$

Using (2.5), we have $\mu_1=1$, $\mu_2=1$, and

$$\Lambda=\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix},$$

which is invertible. Then, we choose the interactor as follows

$$L(s)=\begin{pmatrix} s+2 & 0 \\ 0 & s+2.5 \end{pmatrix}.$$

We now calculate the dummy gain F using (2.10):

$$F=\begin{pmatrix} 5 & -2 & -2 \\ -3 & 2 & 3 \end{pmatrix},$$

The resulting $Q(s)$ using *lemma 1* is

$$Q(s)=\begin{pmatrix} \dfrac{s^2+5s-18}{s^2+7s+10} & \dfrac{2s+19}{s^2+7.5s+12.5} \\ \dfrac{-2s^2-9s-7}{s^2+7s+10} & \dfrac{s^2+4s+1}{s^2+7.5s+12.5} \end{pmatrix}.$$

It can be verified that $P(s)Q(s)=W(s)=L^{-1}(s)$.

In the later development, since the real A,B,C of the system are assumed to be unknown, we will adjust $Q(s)$ by means of FEL and linear parameterization of the feedforward control. At first, the main idea will be presented for SISO in the next chapter. Secondly, we will present how to generalize the SISO case to MIMO system in chapter IV.

Chapter 3
Feedforward Learning Control

3.1 Introduction

The origin of learning feedforward control can be traced back to a scheme known as Feedback Error Learning (FEL). A perfect feedforward can be realized if the feedforward controller provides an inverse of the plant as we derived in the previous section. However, the plant is usually unknown. Thus, we need to obtain the inverse by means of learning feedforward controller.

Basically, there are three broad approaches for learning the inverse of the plant. The first scheme is called a direct approach or called direct inverse modeling [26]. The second approach is an indirect approach or referred as distal supervised learning [27]. The third one is the feedback error learning which combines the aspects of the direct and indirect approaches [1]. All three approaches acquire an inverse of the plant based on samples of inputs and outputs from the plant. Whereas the direct inverse modeling uses these samples to train the inverse directly, the distal supervised learning approach trains the inverse model indirectly, through the intermediary of a learned forward model of the plant. Also, the feedback error learning approach trains the inverse model directly, but it makes use of a feedback error signal as a learning signal.

3.2 Direct Inverse Modeling:

Direct inverse modeling treats the problem of learning an inverse model as a classical supervised learning problem [25]. It is the simplest approach for learning the inverse of the plant as illustrated in Fig. 3.1. The plant receives the input $u(t)$ and produces the output $y(t)$. The feedforward controller is oriented in the output-input direction opposite to that of the plant. Thus, the

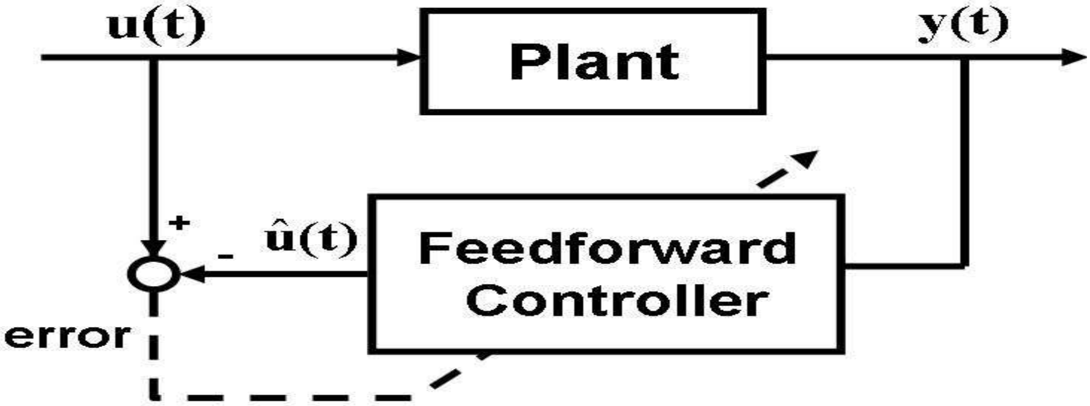


Fig. 3.1 Direct Inverse Modelling Scheme

feedforward controller receives the output of the plant as an input and outputs $\hat{u}(t)$. The error signal is given as the difference between the actual input and the estimated one. The error signal will adjust the inverse model.

Although direct inverse modeling has been shown to be a visible technique in a number of fields [25], [27], it has two main drawbacks which limit its usefulness [26]. First, if the plant is characterized by multi-input to single-output (many-to-one), then the direct inverse modeling technique may be unable to find an inverse. The difficulty is that nonlinear many-to-one mappings can yield nonconvex inverse images which are problematic for the direct inverse modeling. The second drawback is that there is no direct way to find an input that corresponds to a particular desired output.

3.3 Distal Supervised Learning

Distal supervised learning (DSL) approach is proposed by Jordan and Rumelhart in 1992 [26]. This approach combines two models: a forward model and an inverse model as shown in Fig. 3.2. Some of the learning algorithms require access to the internal structure of the plant that they are training. However, the internal structure of the plant is usually unknown. The essence of DSL is that it uses an internal forward model of the plant rather than the plant itself. The forward model is a mapping from states $\hat{x}(t)$ and inputs $u(t)$ to predicted plant output $\hat{y}(t)$ and it is trained using the predicted error ($y(t)-\hat{y}(t)$) as shown on the top of Fig. 3.2. Then, the feedforwad controller and the forward model are joined together and are considered as a single composite learning system. The composite system is

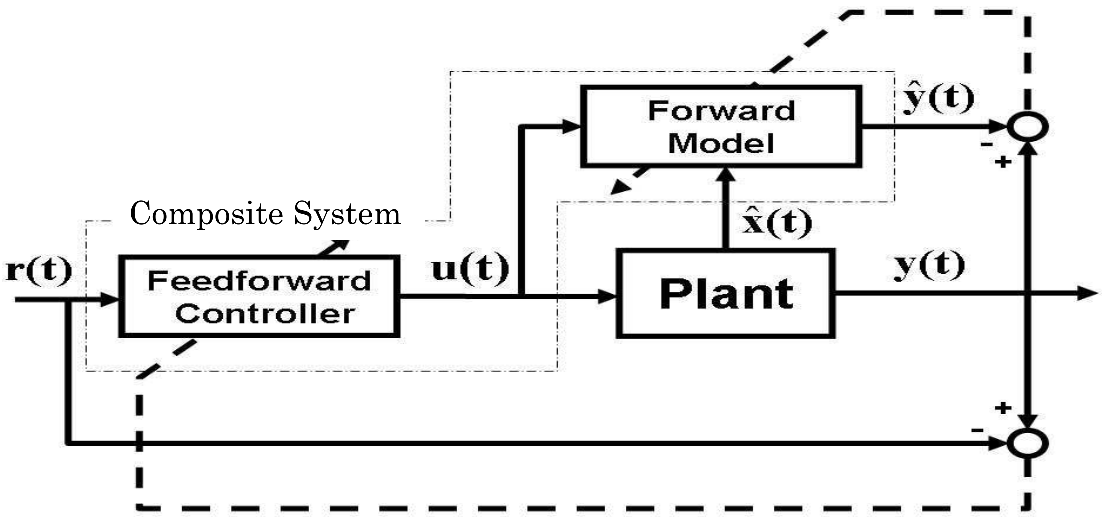


Fig. 3.2 Distal Supervised Learning

trained using performance error ($y(t)-r(t)$). During this training process, the parameters in the forward model are held fixed. Thus, the composite learning system is trained to be identity transformation by a constrained learning process in which some of the parameters inside the system are held fixed. So, the feedforward controller parameters are only altered.

The main drawback of this technique is that the inaccuracies in the forward model may lead to instabilities of the plant if care is not taken in the control system design. Thus, this technique can not guarantee the closed-loop stability if the forward model is not accurate.

3.4 Feedback Error Learning for SISO

Kawato, Furukawa and Suzuki (1987, [1]) have developed a technique called feedback error learning (FEL). The novelty of FEL lies in its use of feedback error as a teaching signal as shown in Fig. 3.3, which is essentially new in control literature [2], [3]. It makes use of the feedback error signal to guide the learning of the feedforward controller. Thus the feedback error is used as tuning signal of the feedforward controller to achieve the inverse of the plant It should be noted that the plant that have been considered in [1] is biproper SISO while the strictly proper SISO plant has been considered in [2] and [3]. Also, a constant feedback gain has been considered in [1], [2] and [3].

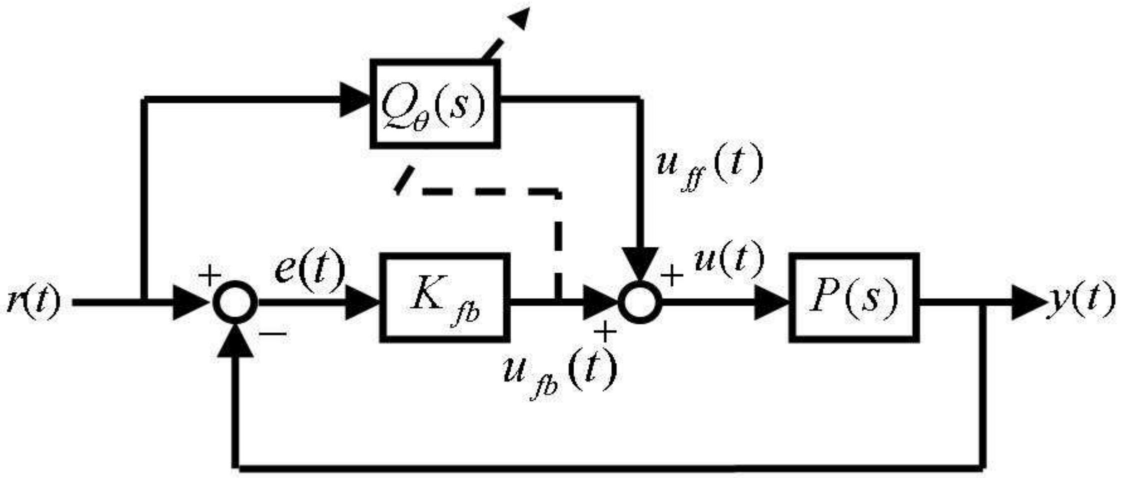


Fig. 3.3 Feedback error learning Architecture

Fig. 3.3 illustrates the architecture of the feedback error learning. The main objective of control is to minimize the error $e(t)$ between the reference signal $r(t)$ and the plant output $y(t)$. The input of the plant $u(t)$ is composed of the output $u_{ff}(t)$ of the tuneable feedforward controller $Q_\theta(s)$ and the output $u_{fb}(t)$ of the feedback controller K_{fb}. If $P(s)$ is known and $P^{-1}(s)$ exists and stable, taking $Q_\theta(s) = P^{-1}(s)$ leads to have perfect tracking. Indeed, this can be verified easily from the relations $u_{ff}(t) = P^{-1}(s)r(t)$, $u_{fb}(t) = K_{fb}(r(t) - y(t))$ and $y(t) = P(s)(u_{ff}(t) + u_{fb}(t))$ derived from the above block diagram that $y(t) = r(t)$. However, $P(s)$ is always unknown. Therefore, we need to obtain the inverse of the plant by means of feedforward learning controller or $Q_\theta(s) \to P^{-1}(s)$ while the feedforward controller $Q_\theta(s)$ will be tuned using feedback error signal.

An important difference between FEL and direct inverse modelling regards the signal used as the controller input. In direct inverse modelling the controller is trained off-line because the input to the controller for the purposes of training is the actual output. However, in order for the controller to participate in the control processes, it has to receive the desired plant output or the reference input as its input. Thus, the direct inverse modelling requires a switching process which means the desired output must be switched in for the purposes of control and the actual output must be switched in for the purposes of training.

The FEL approach provides a more elegant solution for this problem. In FEL, the desired plant output or reference input is used for both training and control. Furthermore, the feedforward controller is trained on-line which means it is used as a controller while it is being trained. This is the main advantage of using feedback error learning to learn the inverse model of the plant. Thus, the feedforward controller converges to an inverse model of the plant because of the error correcting properties of the feedback controller. Moreover, by using the feedback error signal this will increase the robustness of the system.

3.4.1 Analysis of FEL

The main objective of FEL is to implement the feedforward controller as a function approximator, instead of designing linear control on the basis of a plant model. The types of function approximators used in [1], [6] as an example are a Multi Layer Perception (MLP) and Cerebellar Model Articulation Controller (CMAC) neural networks, respectively. Thus, the learning behavior can be improved by selecting different inputs for the function approximator. Gomi and Kawato in [11] have added the error signal as an input to the function approximator which changes the learning controller from a pure feedforward controller into a feedforward and feedback controller. Thus, the tuning signal will be equal to the summation of feedback and feedforward signal.

Several problems exist for this kind of function approximator such as slow convergence, local minima, computational load and high memory requirement [7]. Recently, Miyamura and Kimura [2] have established a control theoretical validity of the FEL method in the frame of adaptive control, proving its stability based on strictly positive realness, whereas Muramatsu and Watanabe [3] have relaxed the positive realness condition of FEL by using the error signal between the reference and the output signal as well as the feedback input. They have used an adaptive linear filter for function approximator, and did not use MLP nor CMAC neural networks any more, but still the learning of the inverse model has been effectively used in [2], [3].

It is assumed in [2] that a true parameter θ_0 exists such that

$$P^{-1}(s) = Q_{\theta_0}(s). \tag{3.1}$$

The learning law is derived using the gradient descent method with the following error function

$$V(t) = \frac{1}{2}(u_{ff}(t) - u_0)^T (u_{ff}(t) - u_0), \tag{3.2}$$

where, u_0 denotes the correct input

$$u_0 = P^{-1}(s) r(t). \tag{3.3}$$

Then, the gradient method yields

$$\frac{d\theta}{dt} = -\alpha \frac{\partial V}{\partial \theta}, \tag{3.4}$$

where α is a scalar parameter to adjust the adaptation speed. Thus, the resulting tuning rule is

$$\frac{d\theta}{dt} = -\alpha \frac{\partial u_{ff}}{\partial \theta}(u_{ff}(t) - u_0). \tag{3.5}$$

However, the above tuning rule can not be implemented due to u_0 is unknown. If we assume that the inverse of the plant exists, then we can take

$$u(t) = u_0. \tag{3.6}$$

As a result, the tuning rule becomes

$$\frac{d\theta}{dt} = \alpha \frac{\partial u_{ff}}{\partial \theta} u_{fb}, \tag{3.7}$$

which can be implemented. The above tuning rule is the core of the FEL and it shows clearly that it depends on the feedback error. It is also clear that the derived learning law depends on the above approximation crucially. This approximation is justified in [2], [3] by proving the stability of the derived learning law.

3.4.2 Stability of FEL

Recently, Miyamura and Kimura [2] considered the stability of FEL for invertible and noninvertible SISO plant and established a control theoretical validation. They studied FEL controller in the framework of adaptive control theory, and proved the stability of FEL algorithm based on the well-known notion of strict positive realness.

A transfer function $G(s)$ is said to be positive real if and only if $\mathrm{Re}[G(s)] \geq 0$, for each $\mathrm{Re}[s] \geq 0$ $\{s = \alpha + jw,\ \alpha \geq 0\}$, where $\mathrm{Re}[.]$ denotes the real part. A transfer function $G(s)$ is said to be strictly positive real if and only if there exists a positive number ε such that $G(s-\varepsilon)$ is positive real [10]. The positive realness condition has been one of the major tools of proving the stability in the literature of adaptive control.

The main result in [2] is that the FEL algorithm is stable and the error $e(t)$ tends to 0 by choosing sufficiently large positive constant feedback gain provided that the actual plant is stable. Furthermore, if the state of the feedforward controller satisfies the so-called persistent excitation (PE) condition [10], then the feedforward controller $Q(s)$ tends to $P^{-1}(s)$ if $P(s)$ is invertible, or tends to $P^{-1}(s)W(s)$ if $P(s)$ is not invertible, where $W(s)$ is a pre-filter to compensate the improperness of the plant.

On the other hand, the stability of FEL algorithm for SISO has been proved by Muramatsu and Watanabe [3] without recourse to positive realness condition. The strictly positive realness in [2] is imposed on a certain transfer function which includes the feedback controller and the unknown parameters of the feedforward controller model. The condition can be satisfied if the feedback controller is chosen sufficiently large positive constant as mentioned above. However, it is not clear that how large the feedback controller gain should be in order to satisfy the positive realness condition because the magnitude of the feedback gain also depends on the unknown parameters of the feedforward controller. Thus, it has been proposed in [3] that the error signal $e(t)$ and the feedback input have been effectively utilized for learning the inverse of the model. By using error signal, the adaptive law without recourse to the positive realness condition has been derived.

Chapter 4
Feedback Error Learning for MIMO Systems

4.1 Introduction

This chapter is devoted to generalization of the feedback error learning to MIMO systems, which is one of the main contributions of this thesis. The proposed architecture is shown in Fig. 4.1. The objective of the control is to

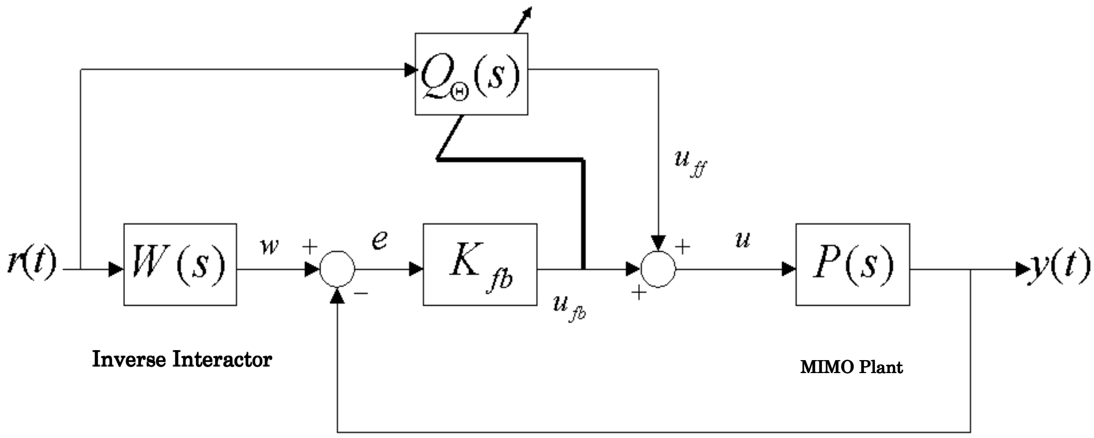


Fig. 4.1 MIMO Feedback Error Learning Architecture

minimize the error signal between $w(t)$ and the plant output $y(t)$. We first note that, if the system is invertible with properness as in [2], [3], then we can readily take $Q_\Theta(s) = P^{-1}(s)$. This is not the case, however, since we have assumed $P(s)$ is strictly proper, and hence $P^{-1}(s)$ becomes improper. We have already derived the ideal feedforward based on the inverse interactorization in section 2.3. However, since the plant parameters are always unknown, we need to derive an MIMO adaptive feedforward control using simple linear parameterization and obtain its tuning rule by generalizing FEL for MIMO systems which will be shown on the next section.

4.2 Adaptive Feedforward

In this section, we establish an adaptive method to adjust the parameter Θ in $Q_\Theta(s)$ when A, B, C, D of the plant are assumed to be unknown. Thus, we generalize FEL for a MIMO system by using a method similar to the adaptive observer parameterization [10]. First, we consider the following dynamical system to parameterize the feedforward controller.

$$
\begin{aligned}
&\dot{\eta}_1(t) = A_f \eta_1(t) + B_f r(t) \\
&\dot{\eta}_2(t) = A_f \eta_2(t) + B_f u_0(t) \\
&u_0(t) = F_0 \eta_1(t) + G_0 \eta_2(t) + H_0 r(t) = \Theta_0 \eta(t),
\end{aligned}
\tag{4.1}
$$

where

$$
\Theta_0 = [F_0 \quad G_0 \quad H_0], \tag{4.2}
$$

$$
\eta(t) = \begin{pmatrix} \eta_1(t) \\ \eta_2(t) \\ r(t) \end{pmatrix}. \tag{4.3}
$$

$F_0, G_0, and\ H_0$ are unknown matrices to be estimated. A_f is any stable matrix and B_f is any matrix such that (A_f, B_f) is controllable.

We will show that the parameterization of (4.1) can yield an arbitrary transfer matrix from $r(t)$ to $u_0(t)$. Consider A_f and B_f in a controllable canonical form as follows:

$$
A_f = \begin{pmatrix}
\begin{matrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ & & \cdots & 1 \\ -f_\mu & & & -f_1 \end{matrix} & & \mathbf{0} \\
& \ddots & \\
\mathbf{0} & & \begin{matrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ & & \cdots & 1 \\ -f_\mu & & & -f_1 \end{matrix}
\end{pmatrix}, \quad
B_f = \begin{pmatrix}
\begin{matrix} 0 \\ \vdots \\ 1 \end{matrix} & & \mathbf{0} \\
& \ddots & \\
\mathbf{0} & & \begin{matrix} 0 \\ \vdots \\ 1 \end{matrix}
\end{pmatrix}. \tag{4.4}
$$

Thus, the transfer matrix from $r(t)$ to $u_0(t)$ is calculated by taking the Laplace transform of (4.1). Therefore, we have

$$u_0(s) = F_0(sI - A_f)^{-1} B_f r(s) + G_0(sI - A_f)^{-1} B_f u_0(s) + H_0 r(s) \tag{4.5}$$

Hence

$$u_0(s) = (I - G_0(sI - A_f)^{-1} B_f)^{-1}(H_0 + F_0(sI - A_f)^{-1} B_f) r(s)\,. \tag{4.6}$$

Taking into account that

$$(sI - A_f)^{-1} B_f = S(s) \frac{1}{s^\mu + f_1 s^{\mu-1} + \cdots + f_\mu} I_m\,, \tag{4.7}$$

where

$$S(s) = \begin{pmatrix} \begin{matrix} 1 \\ s \\ \vdots \\ s^{\mu-1} \end{matrix} & & 0 \\ & \ddots & \\ 0 & & \begin{matrix} 1 \\ s \\ \vdots \\ s^{\mu-1} \end{matrix} \end{pmatrix}, \tag{4.8}$$

we thus obtain

$$u_0 = ((s^\mu + f_1 s^{\mu-1} + \cdots + f_\mu) I - G_0 S(s))^{-1}((s^\mu + f_1 s^{\mu-1} + \cdots + f_\mu) H_0 + F_0 S(s)) r(t)\,. \tag{4.9}$$

It can be shown that equation (4.9) gives a complete representation of any transfer matrix of this size. Namely, by taking F_0, G_0, H_0 appropriately, we can obtain $Q(s)$ derived in the section 2.3, in the form of (4.9). In reality, however, we do not know such parameters $F_0, G_0, and\ H_0$, so that we regard these matrices as tuneable coefficients and adjust them accordingly to some learning law. In this sense we also call (4.9) as an ideal feedforward input.

We now construct a tuneable feedforward controller. In order to generate $u_{ff}(t)$, we consider the following dynamic system, instead of (4.1):

$$
\begin{aligned}
\dot{\xi}_1(t) &= A_f \xi_1(t) + B_f r(t) \\
\dot{\xi}_2(t) &= A_f \xi_2(t) + B_f u(t) \\
u_{ff}(t) &= F(t)\xi_1(t) + G(t)\xi_2(t) + H(t)r(t) = \Theta(t)\xi(t),
\end{aligned}
\tag{4.10}
$$

where

$$\Theta(t) = [F(t) \quad G(t) \quad H(t)], \tag{4.11}$$

$$\xi(t) = \begin{pmatrix} \xi_1(t) \\ \xi_2(t) \\ r(t) \end{pmatrix}. \tag{4.12}$$

Note that the unknown parametric matrices $F(t), G(t), H(t)$ enter linearly. The matrix $\Theta(t)$ is tuned using the learning law, which will be derived in the next section.

4.3 MIMO-FEL Learning Law

The learning law is derived using the gradient method with error function, defined for each time as follows:

$$V = \frac{1}{2}(u_{ff} - u_0)^T (u_{ff} - u_0). \tag{4.13}$$

Then we may think that the steepest gradient method is formulated as

$$\frac{d\Theta}{dt} = -\alpha \frac{\partial V}{\partial \Theta}, \tag{4.14}$$

for some scalar parameter α to adjust the adaptation speed.

A main difference from [2] and [3] is, however, that Θ is not a vector, but a matrix. Still the idea carries over only with a slight modification.

We define

$$\frac{\partial V}{\partial \Theta} = \left(\frac{\partial V}{\partial \theta_{ij}} \right)_{i,j}, \tag{4.15}$$

where

$$\Theta = (\theta_{ij})_{i,j}. \tag{4.16}$$

Thus, we calculate

$$\frac{\partial V}{\partial \theta_{ij}} = \frac{\partial}{\partial \theta_{ij}} \left(\frac{1}{2} \sum_k v_k{}^2 \right), \tag{4.17}$$

where

$$v = u_{ff} - u_0 = (v_1, \cdots, v_m)^T. \tag{4.18}$$

Using (24), we have

$$v = \Theta \xi - u_0. \tag{4.19}$$

Therefore

$$\frac{\partial V}{\partial \theta_{ij}} = \sum_k \left(v_k \frac{\partial v_k}{\partial \theta_{ij}} \right) = \sum_k \left(v_k \frac{\partial}{\partial \theta_{ij}} \left(\sum_l (\theta_{kl} \xi_l - u_{0k}) \right) \right). \tag{4.20}$$

Also, we have

$$\frac{\partial}{\partial \theta_{ij}} \left(\sum_l (\theta_{kl} \xi_l - u_{0k}) \right) = \delta_{ik} \xi_j, \quad \text{where } \delta_{ik} = \begin{cases} 0 & i \neq k \\ 1 & i = k \end{cases}. \tag{4.21}$$

Hence, we have

$$\frac{\partial V}{\partial \theta_{ij}} = v_i \xi_j, \qquad i, j = 1, \cdots, m. \tag{4.22}$$

Therefore, we have

$$\frac{\partial V}{\partial \Theta} = v \xi^T. \tag{4.23}$$

We derived (2.16) as $e(t) = 0$, which implies that we can take $u(t) = u_0(t)$. Therefore, the tuning rule turns out to be

$$\frac{d\Theta}{dt} = -\alpha (u_{ff}(t) - u(t)) \xi^T(t) = \alpha u_{fb}(t) \xi^T(t), \tag{4.24}$$

where α is a scalar parameter introduced to adjust adaptation speed. The above learning law clearly depends on the feedback error signal and that's why it is called feedback error learning. In the next chapter, we will study the stability of the above differential equation.

Chapter 5
Stability of MIMO-FEL Learning Law

5.1 Introduction

Online learning algorithms such as FEL provide a powerful set of tools for automatically fine-tuning of a feedforward controller to improve the performance while in operation, or to automatically adapt the change of dynamics. For FEL to be widely adopted for applications such as airplane flight, it is critical that we provide safety guarantees, specifically stability guarantees. Therefore, stability theory plays a major role in systems theory and engineering.

Often, the mathematical theory of stability analysis is (mis)understood as the process of finding a solution for the differential equation(s) that govern the system dynamics. Stability analysis is the theory of validating the existence (or non-existence) of stable states. Theoretically, there is no guarantee for the existence of a solution to an arbitrary set of nonlinear differential equations [20].

It is often seen that adaptive systems that are stable under some definitions of stability tend to become unstable under other definitions [10]. This difficulty in imposing strong stability restrictions for nonlinear systems was realized as early as a century ago by a Russian mathematician A. M. Lyapunov. In the next section, we will give an overview of this stability theorem.

5.2 Lyapunov Stability Theory

In this section we review the tools of Lyapunov stability theory. These tools will be used in the next section to analyze the stability of FEL. We present a survey of the results that we shall need in the sequel, with no proofs. The interested reader should consult a standard text, such as Khalil [18] or Sastry [20], for details.

5.2.1 Basic Definitions

We consider nonlinear time-invariant system $\dot{x}(t) = f(x)$, where $f : R^n \to R^n$. A point $x_e \in R^n$ is an equilibrium point of the system if $f(x_e) = 0$. Thus, x_e is the equilibrium point if and only if $x(t) \equiv x_e$ is a trajectory. Suppose x_e is an equilibrium point, then we say

- system is globally asymptotically stable (G.A.S.) if for every trajectory $x(t)$, we have $x(t) \to x_e \ as \ t \to \infty$ (implies x_e is the unique equilibrium point).
- system is locally asymptotically stable (L.A.S.) near or at x_e if there is an $R > 0 \ s.t. \ \|x(0) - x_e\| \le R \Rightarrow x(t) \to x_e \ as \ t \to \infty$.
- We can often change the coordinate so that $x_e = 0 \ (i.e., we \ use \ \tilde{x} = x - x_e)$.
- A linear system $\dot{x}(t) = Ax(t)$ is G.A.S. (with $x_e = 0$) $\Leftrightarrow$ $\mathrm{Re}[\lambda_i(A)] < 0, i = 1, \cdots, n$, where $\lambda_i(A)$ is an eigenvalue.
- A linear system $\dot{x}(t) = Ax(t)$ is L. A. S. (near $x_e = 0$) $\Leftrightarrow$ $\mathrm{Re}[\lambda_i(A)] < 0, i = 1, \cdots, n$. (Hence for linear systems L.S.A. $\Leftrightarrow$ G.S.A)

There are many other variants on stability (e.g., uniform stability, exponential stability, . . .) which can be seen in [20].

Positive definite functions:

A function $V : R^n \to R$ is positive definite (pd) if

- $V(z) > 0$ for all $z \ne 0$,
- $V(z) = 0$ if and only if $z = 0$,
- All sublevel sets of V are bounded.

The last condition is equivalent to $V(z) \to \infty \ as \ z \to \infty$.

For example: $V(z) = z^T P z$, with $P = P^T$, is pd if and only if $P > 0$.

5.2.2 Lyapunov theory

Lyapunov theory is used to make conclusions about stability of a system $\dot{x}(t) = f(x)$ (e.g., G.A.S.) without finding the trajectories (i.e., solving the differential equation). A typical Lyapunov theorem has the form:

- ***If*** there exists a function $V : R^n \to R$ that satisfies some conditions on $V \ and \ \dot{V}$,
- ***then*** trajectories of system satisfy some properties concerning stability.

If such a function V exists we call it a Lyapunov function (that proves the property holds for the trajectories). Furthermore, Lyapunov function V can be thought of as a generalized energy function for the system. The main Lyapunov theorems for stability and its application to the linear time invariant case by LaSalle are stated as follows:

1. A Lyapunov boundedness theorem:

Suppose the function V satisfies

- all sublevel sets of V are bounded,
- $\dot{V}(z) \leq 0$ for all z.

Then, all trajectories are bounded, i.e., for each trajectory x there is an R such that $\|x(t)\| \leq R$ for all $t \geq 0$. In this case, V is called a Lyapunov function (for the system) that proves the trajectories are bounded.

2. Lyapunov global asymptotic stability theorem:

Suppose there is a function V such that

- V is positive definite,
- $\dot{V}(z) < 0$ for all $z \neq 0, \dot{V}(0) = 0$.

Then, every trajectory of $\dot{x}(t) = f(x)$ converges to zero as $t \to \infty$ (i.e., the system is globally asymptotically stable).

The interpretation of the above theorem could be:

- V is positive definite generalized energy function.
- Energy is always dissipated, except at 0.

3. Lyapunov exponential stability theorem:

Suppose there is a function V and constant $\alpha > 0$ such that

- V is positive definite,
- $\dot{V}(z) < -\alpha V(z)$ for all $z \neq 0$.

Then, every trajectory of $\dot{x}(t) = f(x)$ satisfies $\|x(t)\| \leq Me^{-\alpha t/2}$ for some M (this is called global exponential stability (G.E.S.)).

Remark: $\dot{V}(z) < -\alpha V(z)$ gives guaranteed minimum dissipation rate, proportional to energy.

4. LaSalle's theorem:

LaSalle's theorem (1960) allows us to conclude G.A.S. of a system with only $\dot{V}(z) < 0$, along with an observability type condition. We consider $\dot{x}(t) = f(x)$, suppose there is a function $V : R^n \to R$ such that

- V is positive definite.

- $\dot{V}(z) < 0$
- The only solution of $\dot{w} = f(w), \dot{V}(w) = 0$ is $w(t) = 0$ for all $t \geq 0$.

Then, the system $\dot{x}(t) = f(x)$ is G.A.S.

The last condition means that there is no nonzero trajectory that can hide in the "zero dissipation" set. Unlike most other Lyapunov theorems, which can be extended to time-varying systems, Lasalle's theorem requires time-invariance.

We now illustrate the use of the stability theorems given above through the following example.

Example: Linear harmonic oscillator

Consider a damped harmonic oscillator, as shown in Fig. 5.1. The

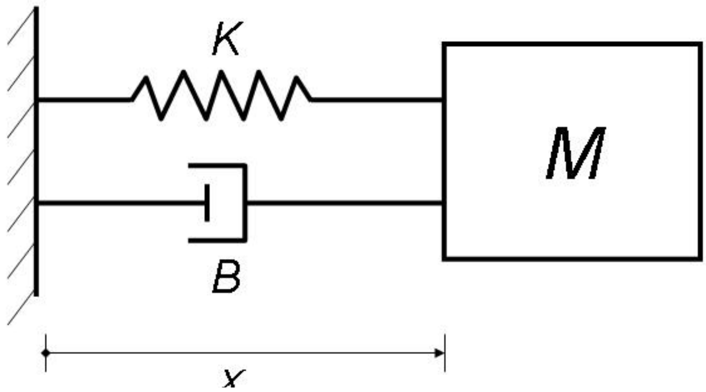


Fig. 5.1 Damped harmonic oscillator.

dynamics of the system are given by the equation

$$M\ddot{x} + B\dot{x} + Kx = 0, \tag{5.1}$$

where M, B, and K are all positive quantities. As a state space equation we rewrite the above equation as

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ -(K/M)x - (B/M)\dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/M & -B/M \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix}. \tag{5.2}$$

Since this system is a linear system, we can determine stability by examining the poles of the system. The A matrix of the system is

$$A = \begin{bmatrix} 0 & 1 \\ -K/M & -B/M \end{bmatrix}, \tag{5.3}$$

and it has the following characteristic equation

$$\lambda^2 + (B/M)\lambda + (K/M) = 0. \tag{5.4}$$

The solutions of the characteristic equations are

$$\lambda = \frac{-B \pm \sqrt{B^2 - 4KM}}{2M}, \tag{5.5}$$

which always have negative real parts, and hence the system is (globally) exponentially stable.

We now try to apply Lyapunov's method to determine exponential stability. The "obvious" Lyapunov function to use in this context is the energy of the system,

$$V(x,t) = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}Kx^2. \tag{5.6}$$

Taking the derivative of V along trajectories of the system (5.6) gives

$$\dot{V} = M\dot{x}\ddot{x} + Kx\dot{x} = -B\dot{x}^2. \tag{5.7}$$

The function $\dot{V}$ is quadratic but not locally positive definite, since it does not depend on x, and hence we cannot conclude exponential stability using Lyapunov theory. However, it is still possible to conclude asymptotic stability using Lasalle's invariance principle which is mentioned above. This verifies the exponential stability which is proved by checking the eigenvalues of A and using Lasalle`s theorem.

5.3 Stability of MIMO–FEL with Positive Realness Condition

By means of interactor concept [14], we have derived in section 2.3 that

$$Q(s) = \left[I - F(sI - A + BF)^{-1}B\right]\Lambda^{-1}, \tag{5.8}$$

thus we have

$$P(s)Q(s) = W(s), \tag{5.9}$$

Therefore, the ideal feedforward control is:

$$u_0(t) = Q(s)r(t)\,. \tag{5.10}$$

We can obtain the following relation

$$\begin{aligned} &W(s)u_0(t) - W(s)u(t) \\ &= W(s)P^{-1}(s)W(s)r(t) - W(s)P^{-1}(s)y(t) \\ &= W(s)P^{-1}(s)[W(s)r(t) - y(t)] \\ &= W(s)P^{-1}(s)e(t). \end{aligned} \tag{5.11}$$

Whereas $W(s)$ works as compensator for $P^{-1}(s)$ because it is not invertible with properness. Since A_f is stable, the comparison between (4.1) and (4.10) by using (5.11) yields the following asymptotic relations

$$\begin{aligned} &\eta_1 \to \xi_1, \\ &\eta_2 \to \xi_2 + (sI - A_f)^{-1} B_f W(s) P(s)^{-1} e(t). \end{aligned} \qquad (as\ \ t \to \infty) \tag{5.12}$$

Substituting (4.1) into (5.11) yields

$$W(s)u(t) = W(s)\Theta_0 \eta(t) - W(s)P^{-1}(s)e(t)\,. \tag{5.13}$$

Substituting (5.12) into (5.13) yields

$$W(s)u(t) = W(s)\Theta_0 \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ r(t) \end{bmatrix} + W(s)\left[-I + G_0(sI - A_f)B_f\right]W(s)P^{-1}(s)e(t). \tag{5.14}$$

Thus, we have

$$u(t) = \Theta_0 \xi(t) + \left[-I + G_0(sI - A_f)B_f\right]W(s)P^{-1}(s)e(t)\,, \tag{5.15}$$

$$but,\ \ u(t) = u_{ff}(t) + K_{fb} e(t)\,. \tag{5.16}$$

Substitute (4.6) and (5.16) in (5.15), we obtain

$$\Theta(t)\xi(t) - \Theta_0 \xi(t) = -K_{fb} e(t) + (-H_0 - F_0(sI - A_f)^{-1} B_f) e(t)\,. \tag{5.17}$$

Therefore, we have

$$\Psi(t)\xi(t) = -\left[K_{fb} + H_0 + F_0(sI - A_f)^{-1}B_f\right]e(t), \tag{5.18}$$

where

$$\Psi(t) := \Theta(t) - \Theta_0. \tag{5.19}$$

As a result, we have

$$e(t) = -\left[K_{fb} + H_0 + F_0(sI - A_f)^{-1}B_f\right]^{-1}\Psi(t)\xi(t). \tag{5.20}$$

Substituting $e(t)$ in the derived learning law (4.24), we obtain

$$\frac{d\Theta}{dt} = -\alpha K_{fb}\left[K_{fb} + H_0 + F_0(sI - A_f)^{-1}B_f\right]^{-1}\Psi(t)\xi(t)\xi^T(t). \tag{5.21}$$

Namely, we have

$$\frac{d\Psi}{dt} = -\alpha K_{fb}\left[K_{fb} + H_0 + F_0(sI - A_f)^{-1}B_f\right]^{-1}\Psi(t)\xi(t)\xi^T(t), \tag{5.22}$$

and let $G(s) = K_{fb}\left[K_{fb} + H_0 + F_0(sI - A_f)^{-1}B_f\right]^{-1}$. (5.23)

As a result, we obtain

$$\frac{d\Psi}{dt} = -\alpha G(s)\Psi(t)\xi(t)\xi^T(t). \tag{5.24}$$

<u>KYP Lemma</u> (Kalman-Yakubovich-Popov)

Let $G(s) := D + C(sI - A)^{-1}B$, (5.25)

be $p \times p$ transfer matrix, where (A,B) is controllable and (A,C) is observable. Then, $G(s)$ is strictly positive real if and only if there exist matrices $P = P^T > 0$, L, and W, and a positive constant ε such that

$$\begin{aligned} &PA + A^T P = -L^T L - \varepsilon P, \\ &PB = C^T - L^T W, \\ &W^T W = D + D^T. \end{aligned} \tag{5.26}$$

For the following differential equation

$$\frac{dZ(t)}{dt} = -\alpha G(s)Z(t)\xi(t)\xi^T(t), \tag{5.27}$$

we derive the following lemma:

Lemma1:

Let $G(s)$ be strictly positive real transfer matrix and $\xi(t)$ be an arbitrary time-varying vector. Then, the solution $Z(t)$ of the above differential equation tends to a constant matrix Z_0 such that $Z_0\xi(t) \to 0$. If $\xi(t)$ satisfies the so-called persistent excitation (PE) condition, the above Z_0 is equal to **0**.

Remark: for PE condition see App. B.

Proof of Lemma 1:

The above differential equation can be written as follows

$$\begin{aligned} &\frac{dx(t)}{dt} = Ax(t) + BZ(t)\xi(t) \\ &y(t) = Cx(t) + DZ(t)\xi(t) \quad . \\ &\frac{dZ(t)}{dt} = -y(t)\xi^T(t) \end{aligned} \tag{5.28}$$

Since $G(s)$ is strictly positive real, it implies that there exist $P = P^T > 0, L$, and W, and a positive constant ε by using KYP lemma. We consider the following Lyapunov function

$$V(t) = tr\left(Z^T(t)Z(t)\right) + x(t)^T Px(t) . \tag{5.29}$$

Taking the derivative

$$\begin{aligned} \dot{V}(t) &= tr\left(Z^T(t)\dot{Z}(t)\right) + tr\left(\dot{Z}^T(t)Z(t)\right) + \dot{x}(t)^T Px(t) + x(t)^T P\dot{x}(t) \\ \dot{V}(t) &= tr\left(Z^T(t)y(t)\xi^T(t)\right) + tr\left(\xi(t)y^T(t)Z(t)\right) + \left(Ax(t) + BZ(t)\xi(t)\right)^T Px(t) + \\ &\quad x(t)^T P\left(Ax(t) + BZ(t)\xi(t)\right) \\ &= \xi^T(t)Z^T(t)y(t) + y^T(t)Z(t)\xi(t) + x^T(t)\left(PA + A^T P\right)x(t) + \\ &\quad x^T(t)PBZ(t)\xi(t) + \xi^T(t)Z^T B^T Px(t) \end{aligned} \tag{5.30}$$

Using KYP lemma, we have

$$\begin{aligned} \dot{V}(t) &= \xi^T(t)Z^T(t)y(t) + y^T(t)Z(t)\xi(t) + x^T(t)\left(-L^T L - \varepsilon P\right)x(t) + \\ &\quad x^T(t)\left(C^T - L^T W\right)Z(t)\xi(t) + \xi^T(t)Z^T\left(C - W^T L\right)x(t) \end{aligned}$$

$$\dot{V}(t) = \xi^T(t)Z^T(t)y(t) + y^T(t)Z(t)\xi(t) - x^T(t)L^T Lx(t) - \varepsilon x^T(t)Px(t) +$$
$$x^T(t)C^T Z(t)\xi(t) - x^T(t)L^T W Z(t)\xi(t) + \xi^T(t)Z^T(t)Cx(t) - \xi^T(t)Z^T(t)W^T Lx(t)$$
$$\dot{V}(t) = -\left|Lx(t) + WZ(t)\xi(t)\right|^2 - \varepsilon x^T(t)Px(t) \le 0 \tag{5.31}$$

Since $V(t) \ge 0$ while $\dot{V}(t) \le 0$, this implies that $x(t)$ and $Z(t)\xi(t)$ converges to 0. Also, as $x(t) \to 0$, we have

$$\frac{dZ(t)}{dt} = -DZ(t)\xi(t)\xi^T(t). \tag{5.32}$$

As a result, if $\xi(t)$ satisfies PE then the above differential equation is globally exponentially stable $Z(t) \to 0$ based on the PE and Exponential Stability Theorem. □

Theorem1:

Under any stable transfer matrix $P(s)$ with minimal phase and known upper bound of the relative degree ($\mu := \max(\mu_1, \cdots, \mu_m)$), the MIMO-FEL scheme and its learning law are stable if K_{fb} is chosen such that $G(s)$ is strictly positive real. Also, it yields $e(t) \to 0$, and $Q_\Theta(s) \to P(s)^{-1}W(s)$ if $\zeta(t)$ satisfies the PE condition.

Proof:

Based on lemma 1, if K_{fb} is chosen such that $G(s)$ is strictly positive real, then the solution of the adaptive law based on FEL will tend to a constant matrix which proves the stability. Also, if $\zeta(t)$ satisfies the PE condition, then $Z(t) \to 0$ and $e(t) \to 0$ as time proceeds and ends up with $Q_\Theta(s) \to P(s)^{-1}W(s)$. □

5.4 Stability of MIMO–FEL without Positive Realness Condition

Now, we establish the stability of the FEL without the positive realness condition. The main idea is to consider $G(s)$ being equal to the identity matrix by influencing sort of dynamic which is based on the error signal. The new vector $\xi_e(t)$ is generated by $e(t)$ with the following dynamics

$$\dot{\xi}_e(t) = A_f \xi_e(t) + B_f e(t), \tag{5.33}$$

where A_f and B_f are as defined before.

From (5.11), (5.12) and (4.6), the second term on the right hand side of (5.15) can be written as

$$\left[-I+G_0(sI-A_f)B_f\right]W(s)P^{-1}(s)e(t)=(-H_0-F_0(sI-A_f)^{-1}B_f)e(t)\,. \tag{5.34}$$

Taking Laplace transform of (5.33) and substitute it into (5.34), we obtain

$$\left[-I+G_0(sI-A_f)B_f\right]W(s)P^{-1}(s)e(t)=(-H_0e(t)-F_0\xi_e)\,. \tag{5.35}$$

Finally, substitute (5.35) into (5.15), and define

$$\zeta(t):=\begin{bmatrix}\xi_1(t)-\xi_e(t)\\ \xi_2(t)\\ W(s)r(t)-e(t)\end{bmatrix}, \tag{5.36}$$

we obtain

$$u(t)=\Theta_0\zeta(t)\,, \tag{5.37}$$

which is linearly parameterized by Θ_0. In order to derive an adaptive law for the FEL, we define $\hat{u}(t)$ by replacing Θ_0 in (5.37) with $\Theta(t)$

$$\hat{u}(t):=\Theta(t)\zeta(t)\,, \tag{5.38}$$

and define an error signal $s(t)$ as

$$\begin{aligned}s(t)&:=u(t)-\hat{u}(t)\\ &=-\psi(t)\zeta(t),\end{aligned} \tag{5.39}$$

where

$$\psi(t):=\Theta(t)-\Theta_0\,. \tag{5.40}$$

Using the gradient method as we did before with error function $s(t)$, the learning law turns out to be

$$\frac{d\Theta}{dt}=\alpha s(t)\zeta(t)^T\,. \tag{5.41}$$

Using (5.39) and (5.40), the learning law can be written as

$$\frac{d\Psi(t)}{dt}=-\alpha\Psi(t)\zeta(t)\zeta(t)^T\,. \tag{5.42}$$

The stability of the above differential equation (5.40) can be verified using the following derived lemma.

Lemma 2:

Let $\tau(t)$ be an arbitrary time-varying vector. Then, the solution of the differential equation

$$\frac{d\mathrm{Z}(t)}{dt} = -\mathrm{Z}(t)\tau(t)\tau(t)^T \tag{5.43}$$

tends to a constant matrix Z_0 such that $\mathrm{Z}_0\tau(t) \to 0$. If $\tau(t)$ satisfies the persistent excitation (PE) condition [10], the above matrix Z_0 is equal to zero matrix.

Proof of Lemma 2:

Let the Lyapunov function defined as follows

$$V(t) = \frac{1}{2} tr\left\{\mathrm{Z}(t)^T \mathrm{Z}(t)\right\}.$$

Then, the derivative along the trajectory is given by

$$\begin{aligned}\dot{V}(t) &= tr\left\{\mathrm{Z}(t)^T \dot{\mathrm{Z}}(t)\right\} \\ &= -tr\left\{\mathrm{Z}(t)^T \mathrm{Z}(t)\tau(t)\tau(t)^T\right\} \leq 0.\end{aligned}$$

Since $V(t) \geq 0$ while $\dot{V}(t) \leq 0$, this implies that the above differential equation is stable according to Lyapunov theorem. Therefore, $Z(t)\tau(t)$ converges to zero vector and thus $\frac{\mathrm{dZ(t)}}{dt} \to 0$. For the second part of lemma 2, if $\tau(t)$ is PE then the learning law is globally exponentially stable, namely $Z(t) \to 0$ based on the PE and Exponential Stability Theorem, Theorem 2.5.1 in [20], see App. C. □

Theorem 2:

Under any stable transfer matrix $P(s)$ with minimal phase and known upper bound of the relative degree ($\mu := \max(\mu_1, \cdots, \mu_m)$), the MIMO-FEL scheme and its learning law

$$\frac{d\Psi(t)}{dt} = -\alpha\Psi(t)\zeta(t)\zeta(t)^T$$

are stable Also, it yields $e(t) \to 0$, and $Q_\Theta(s) \to P(s)^{-1}W(s)$ if $\zeta(t)$ satisfies the PE condition.

Proof:
From lemma 2 $\Psi(t)$ tends to constant matrix Ψ_0 such that $\Psi_0\zeta(t)\to 0$. Therefore, $s(t)=-\Psi(t)\zeta(t)\to 0$. If $\zeta(t)$ satisfies the PE condition then Ψ_0 is equal 0, and thus $\Theta(t)\to\Theta_0$ using (5.40). This implies that $e(t)\to 0$ as time proceeds and ends up with $Q_\Theta(s)\to P(s)^{-1}W(s)$. □

Chapter 6
Simulation Results

To show the effectiveness of the proposed method, three numerical simulation examples have been performed. The three examples correspond to first, second and third order transfer matrices, respectively. At first, we need to tune the adjustable parameter of the feedforward controller $\Theta(t)$ using the learning rule (4.24) so that we can obtain adaptively the inverse of the system in order to track the reference signals.

For each example, we consider three cases, each of which is corresponding to different reference signals. We have considered the following reference inputs $r(t)$: step, sine and random inputs. Furthermore, we fix the feedback gain for each example in order to verify the achievement of the MIMO-FEL by comparing the tracking performance of the outputs using conventional feedback controller versus the output using the proposed MIMO-FEL.

Note: All the numerical simulation examples have been performed using MATLAB (SIMULINK) version 7.01.

6.1 Example 1:

Given

$$A = \begin{pmatrix} -5 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -7 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

we have the following transfer matrix of the plant:

$$P(s) = \begin{pmatrix} \dfrac{1}{s+5} & \dfrac{1}{s+2} \\ \dfrac{1}{s+3} & \dfrac{1}{s+7} \end{pmatrix}.$$

Using (2.5), we have $\mu_1 = 1, \mu_2 = 1$ and thus we set the following diagonal inverse interactor (pre-filter) using (2.8)

$$L^{-1}(s) = W(s) = \begin{pmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+2.5} \end{pmatrix}.$$

Also, we can obtain the upper bound of the relative degree using (2.12) which is equal to $\mu = 1$ in order to know the dimension of the dynamic feedforward system matrices. As a result, we set the following the following A_f and B_f for the dynamic system of the feedforward controller (4.10) based on (4.4)

$$A_f = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}, B_f = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Then, we choose the following feedback gain that maintain the closed-loop stability

$$K_{fb} = \begin{pmatrix} 0.2 & 0.1 \\ 0.4 & 0.1 \end{pmatrix}.$$

After that, we use the derived learning law (4.24) to adjust the tuneable parameter $\Theta(t)$ of the feedforward controller. In order to show the tracking capability of the proposed method, we have considered three different cases of the reference inputs whereas each case corresponds to specific reference signals. The simulation results in Fig. 6.1, Fig. 6.4 and Fig. 6.7 show the outputs $y(t)$ versus $w(t)$ for step, sine and random reference inputs using normal feedback controller without MIMO-FEL, respectively. It can be seen clearly that the tracking performance is not good for all of the three kinds of reference inputs. Thus, the outputs $y(t)$ fail to track their reference signals $w(t)$. However, the simulation results in Fig. 6.2, Fig. 6.5 and Fig. 6.8 show that we have successfully tracked the same reference signals $w(t)$ using MIMO-FEL. This indicates that the feedback controller concerns about stabilizing the system while the feedforward improves the performance of the system. Furthermore, all the components of the tuneable parameters $F(t), G(t), and\ H(t)$ which correspond to the adjustable parameter of the adaptive feedforward controller $\Theta(t)$ tend to constant values as shown in Fig. 6.3, Fig. 6.6 and Fig. 6.9 for step, sine and random reference signals, respectively. Since we have obtained almost perfect tracking for three different reference signals, this concludes that the learning rule successfully learns the inverse of the MIMO system.

Case1: for $r(t)$ = Step Inputs

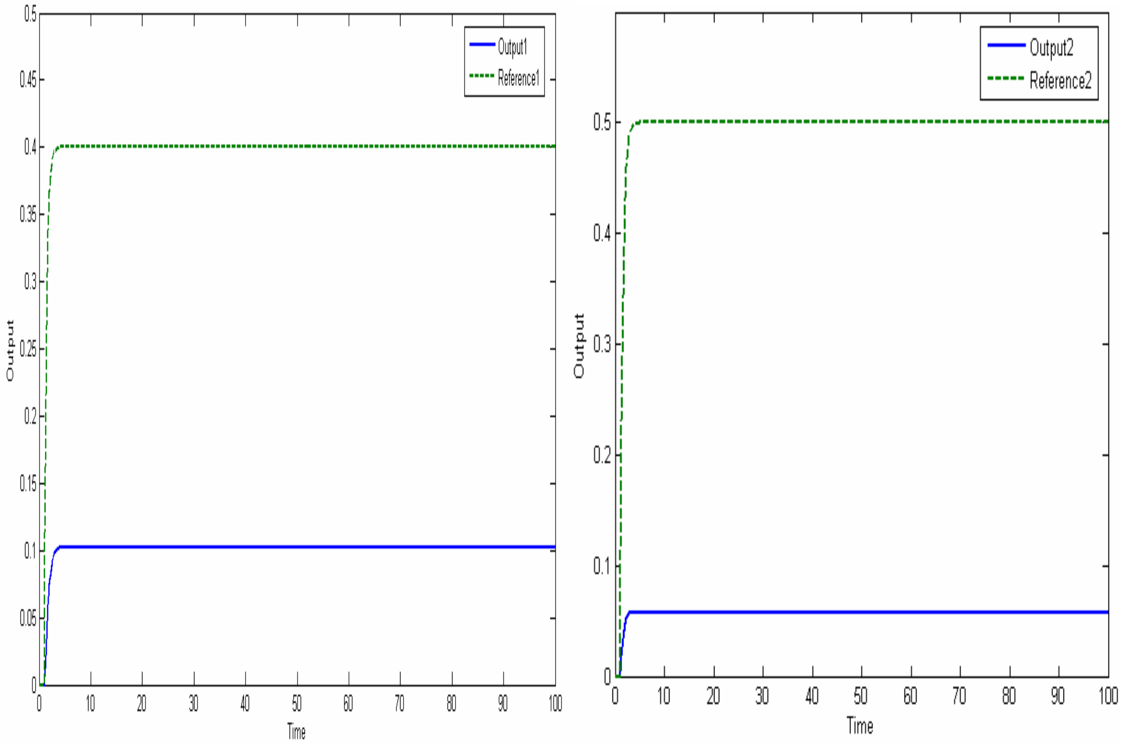


Fig. 6.1 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

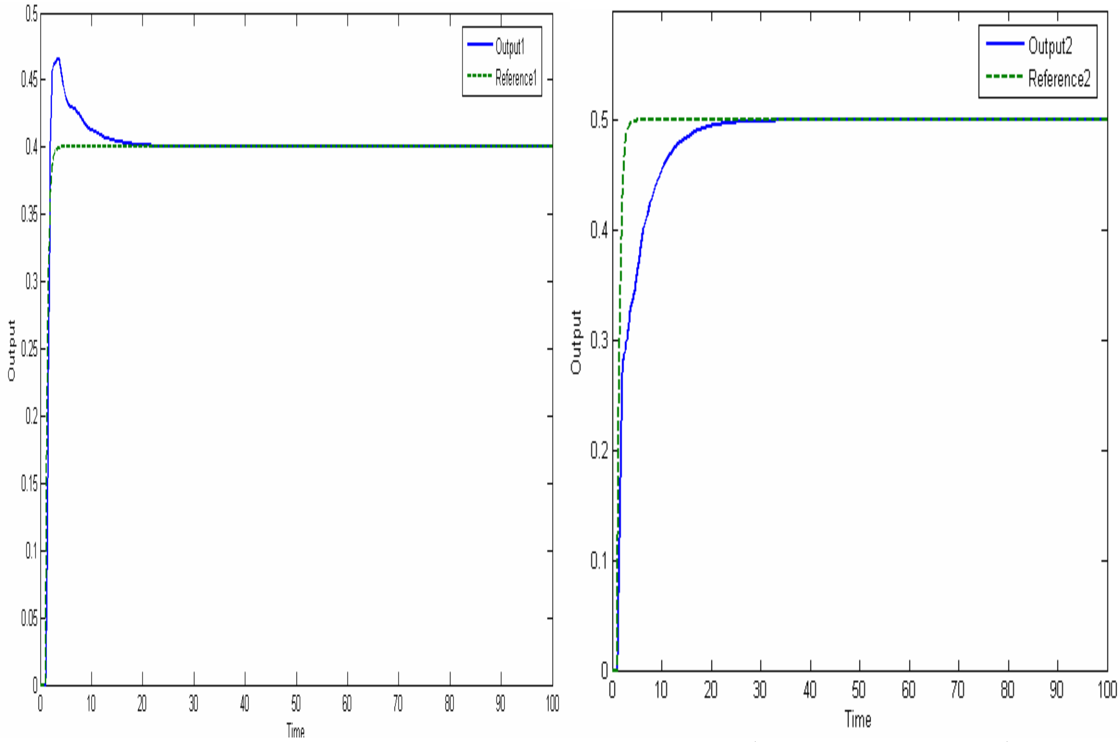


Fig. 6.2 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

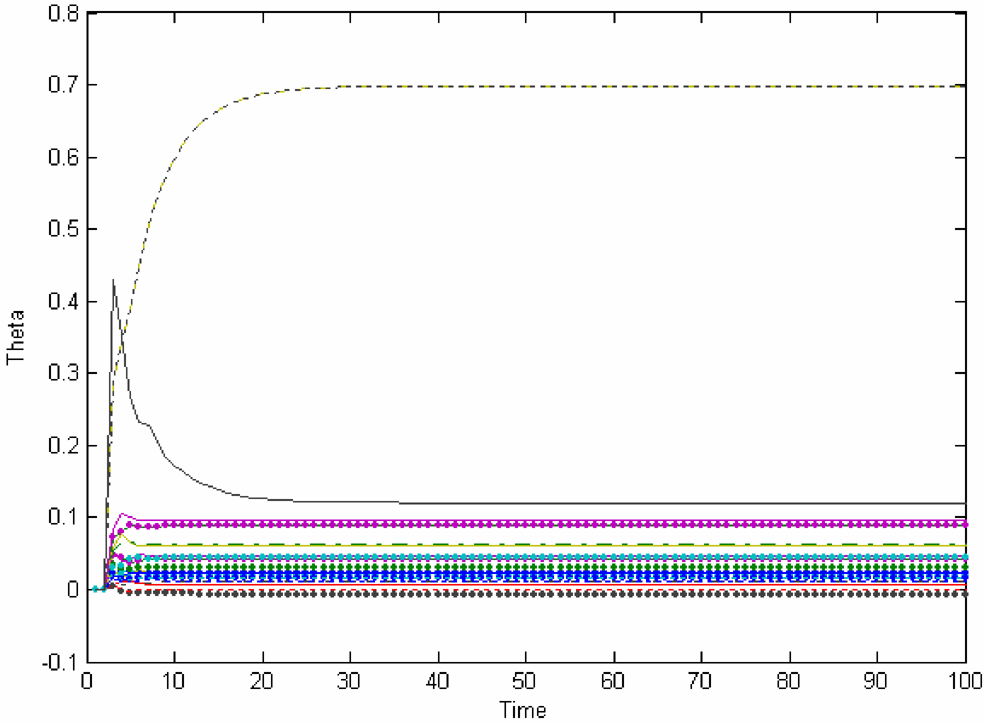


Fig. 6.3 Time Evolution of Tuneable Parameters (Theta)

Case2: for $r(t)$ = Sinusoidal (sin) Signals

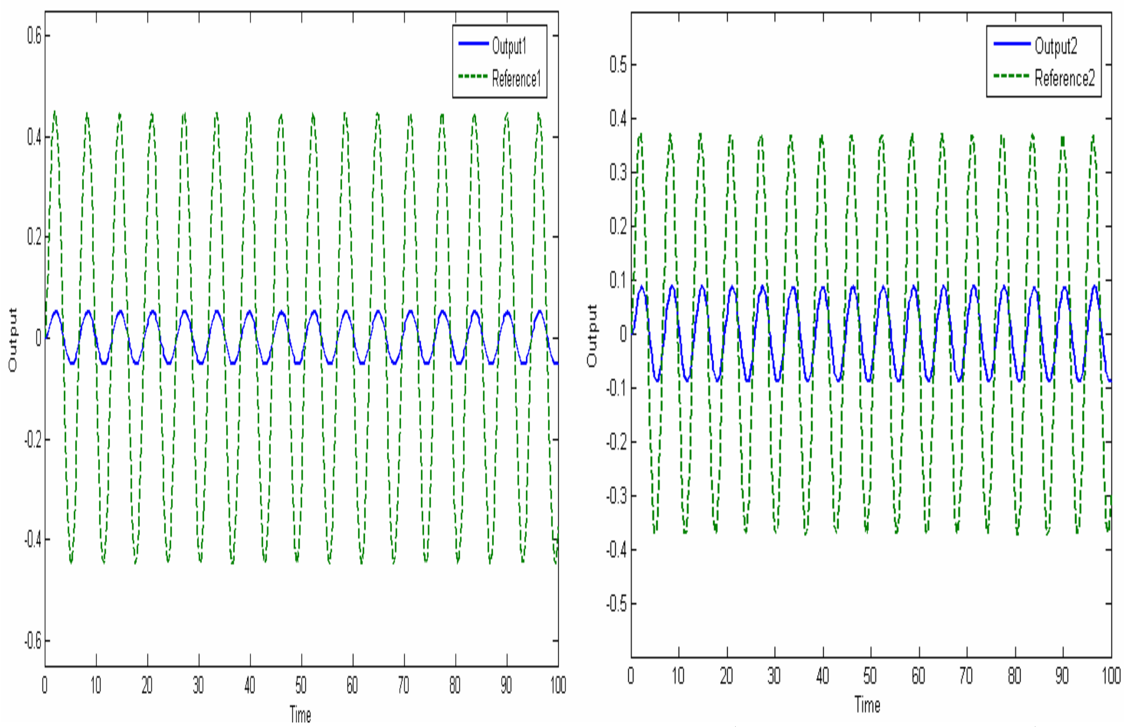


Fig. 6.4 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

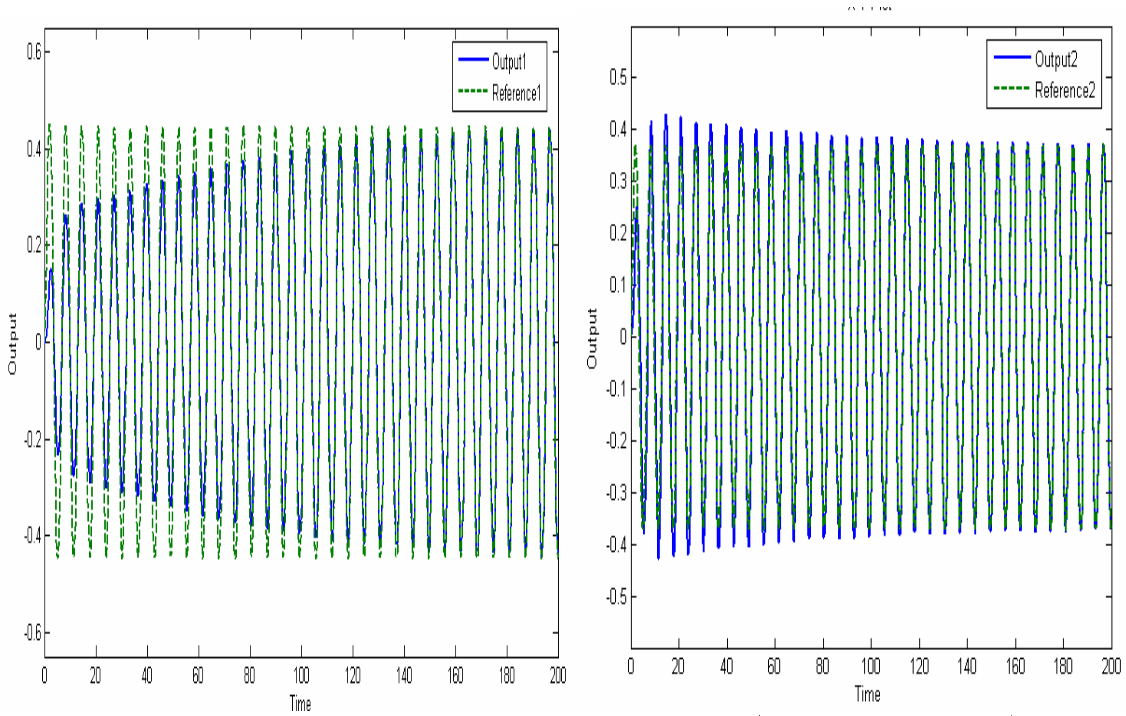


Fig. 6.5 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

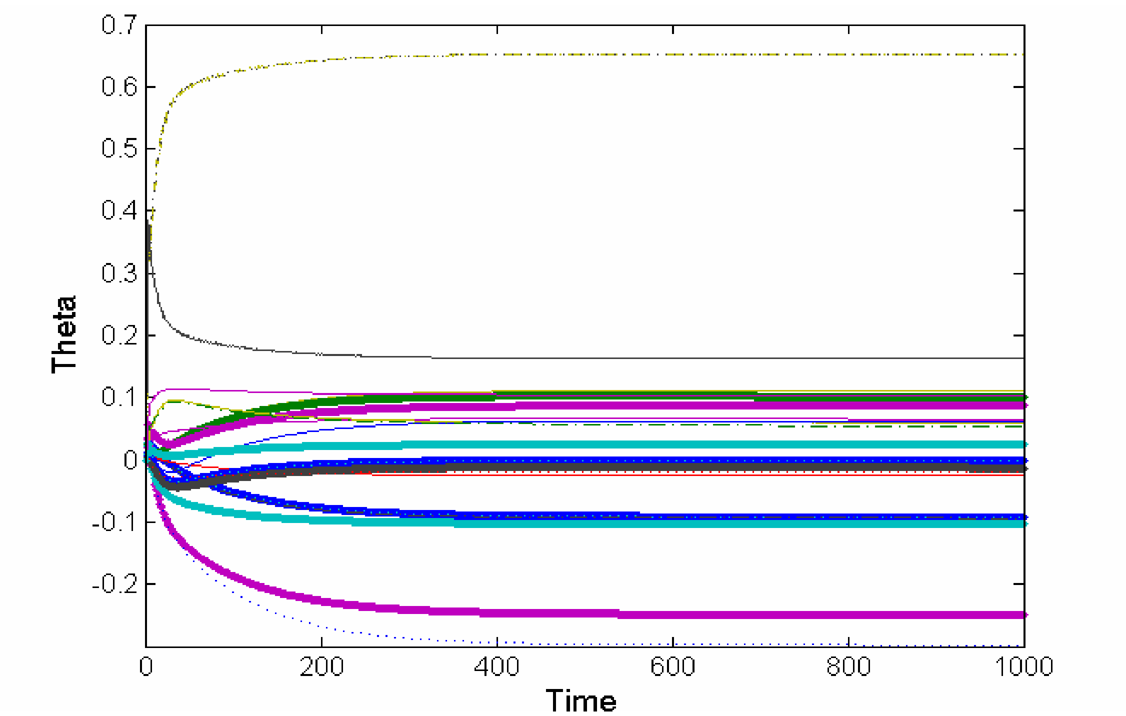


Fig. 6.6 Time Evolution of Tuneable Parameters Theta

Case3: for $r(t)$ = Random Signals

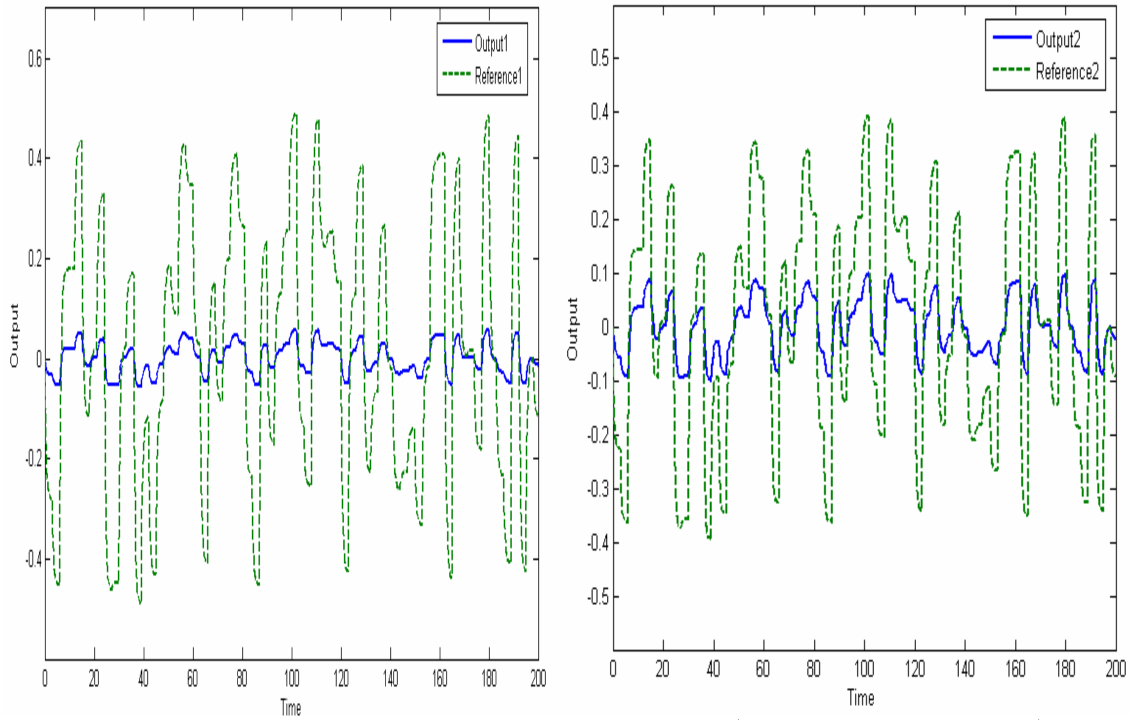


Fig. 6.7 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

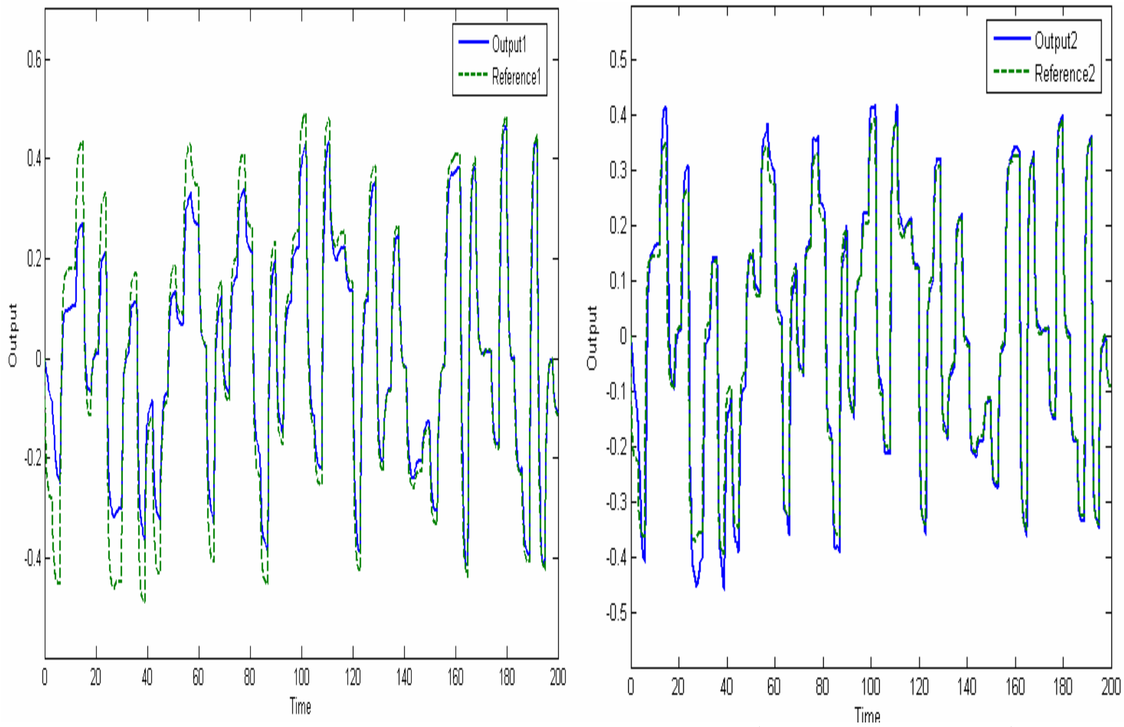


Fig. 6.8 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

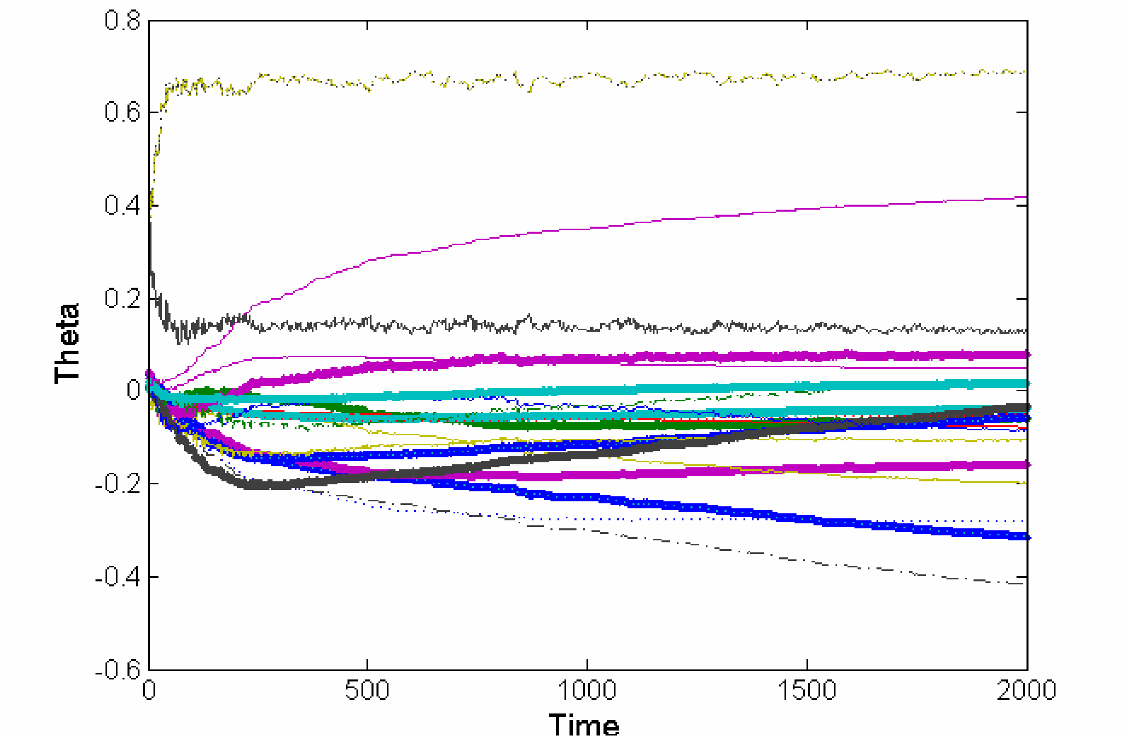


Fig. 6.9 Time Evolution of Tuneable Parameters Theta

6.2 Example 2:

Giving

$$A=\begin{pmatrix}-5 & -3 & 0 & 0\\ 2 & 0 & 0 & 0\\ 0 & 0 & -5 & -3\\ 0 & 0 & 2 & 0\end{pmatrix}, B=\begin{pmatrix}2 & 0\\ 0 & 0\\ 0 & 2\\ 0 & 0\end{pmatrix}, C=\begin{pmatrix}0.5 & 0.25 & 0.5 & 1\\ 0.5 & 1.25 & 0.5 & 1.5\end{pmatrix},$$

we have the following transfer matrix of the plant:

$$P(s)=\begin{pmatrix}\dfrac{s+1}{s^2+5s+6} & \dfrac{s+4}{s^2+5s+6}\\ \dfrac{s+5}{s^2+5s+6} & \dfrac{s+6}{s^2+5s+6}\end{pmatrix}.$$

Using (2.5), we have $\mu_1=1, \mu_2=1$ and thus we set the following diagonal inverse interactor (pre-filter) using (2.8)

$$L^{-1}(s) = W(s) = \begin{pmatrix} \dfrac{1}{s+2} & 0 \\ 0 & \dfrac{1}{s+2.5} \end{pmatrix}.$$

Also, we obtain the upper bound of the relative degree using (2.12) which is equal to $\mu = 1$ in order to know the dimension of the dynamic feedforward system matrices. As a result, we set the following the following A_f and B_f for the dynamic system of the feedforward controller (4.10) based on (4.4)

$$A_f = \begin{pmatrix} -4 & 0 \\ 0 & -5 \end{pmatrix}, \; B_f = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

We choose the following feedback gain matrix that maintains the closed-loop stability

$$K_{fb} = \begin{pmatrix} 0.2 & 0.1 \\ 0.4 & 0.1 \end{pmatrix}.$$

Then, we use the derived learning law (4.24) to adjust the tuneable parameter $\Theta(t)$ of the feedforward controller. In order to show the tracking capability of the proposed method, we have considered three different cases of the reference input whereas each case corresponds to specific reference signals. The simulation results in Fig. 6.10, Fig. 6.13 and Fig. 6.16 show the outputs $y(t)$ versus $w(t)$ for step, sine and random reference inputs using conventional feedback controller without MIMO-FEL, respectively. It can be seen clearly that the tracking performance is not good for all of the three kinds of reference signals. Thus, the outputs $y(t)$ fail to track their reference signals $w(t)$. However, the simulation results in Fig. 6.11, Fig. 6.14 and Fig. 6.17 show that we have successfully tracked the same reference signals $w(t)$ using the proposed MIMO-FEL. This also verifies that the feedback controller concerns about stabilizing the system while the feedforward improves the performance of the system. Furthermore, all the components of the tuneable parameters $F(t), G(t), and\ H(t)$ which correspond to tuneable parameter of the adaptive feedforward controller $\Theta(t)$ tend to constant matrix as shown in Fig. 6.12, Fig. 6.15 and Fig. 6.18 for step, sine and random reference inputs, respectively. Since we have almost obtained perfect tracking of three different cases of the reference inputs, this concludes that the learning rule successfully learns the inverse of the MIMO system.

Case1: for $r(t)$ = Step Inputs

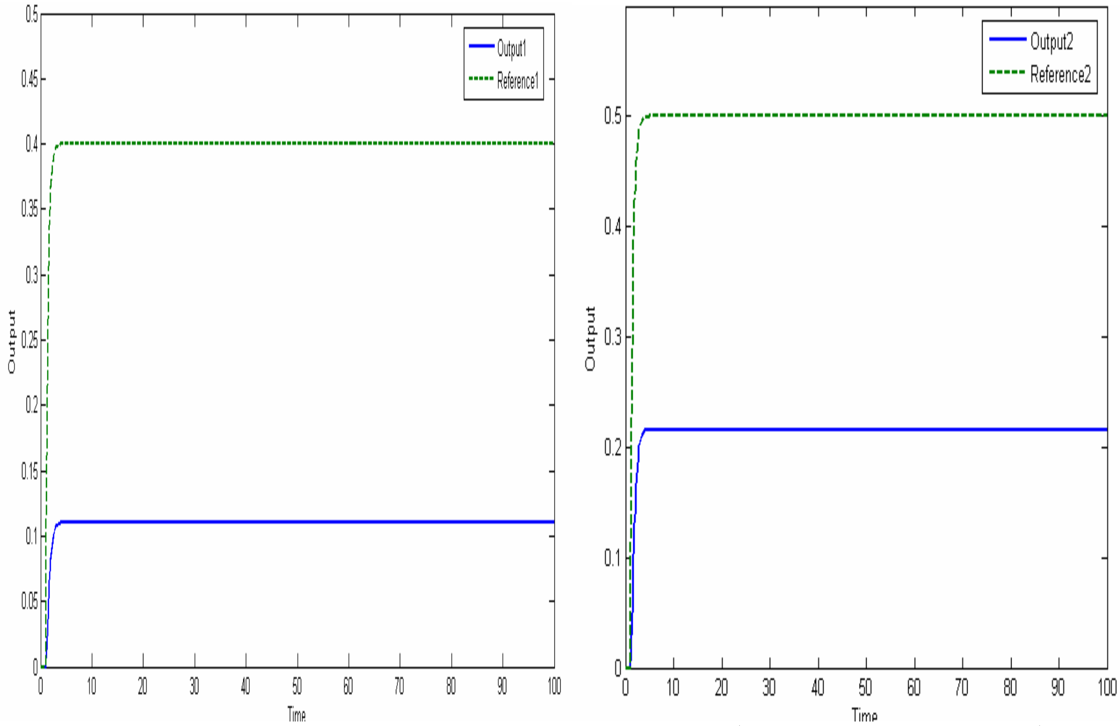


Fig. 6.10 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

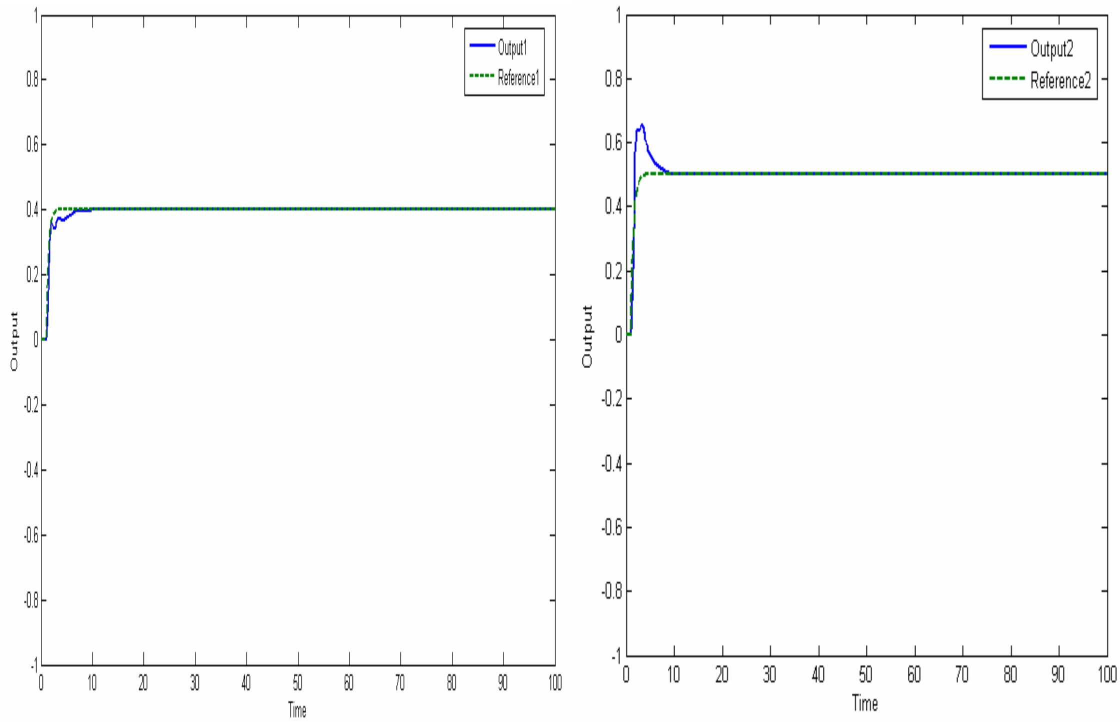


Fig. 6.11 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

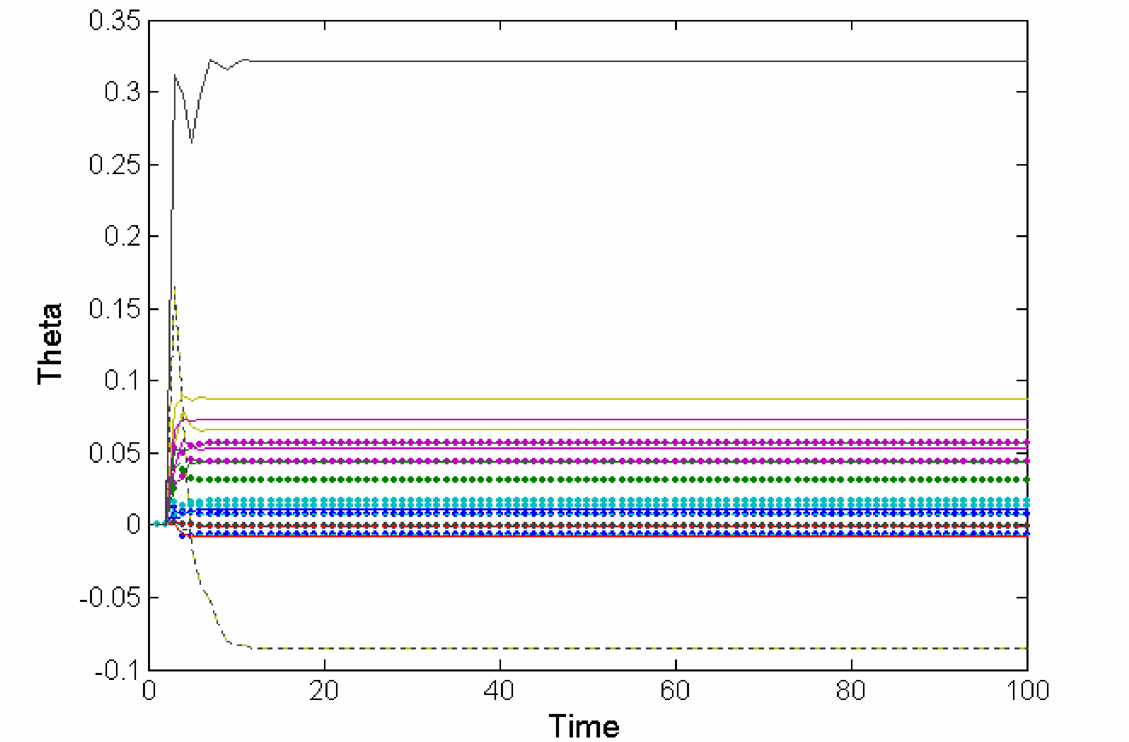


Fig. 6.12 Time Evolution of Tuneable Parameters Theta

Case2: for $r(t)$ = Sinusoidal (sin) Signals

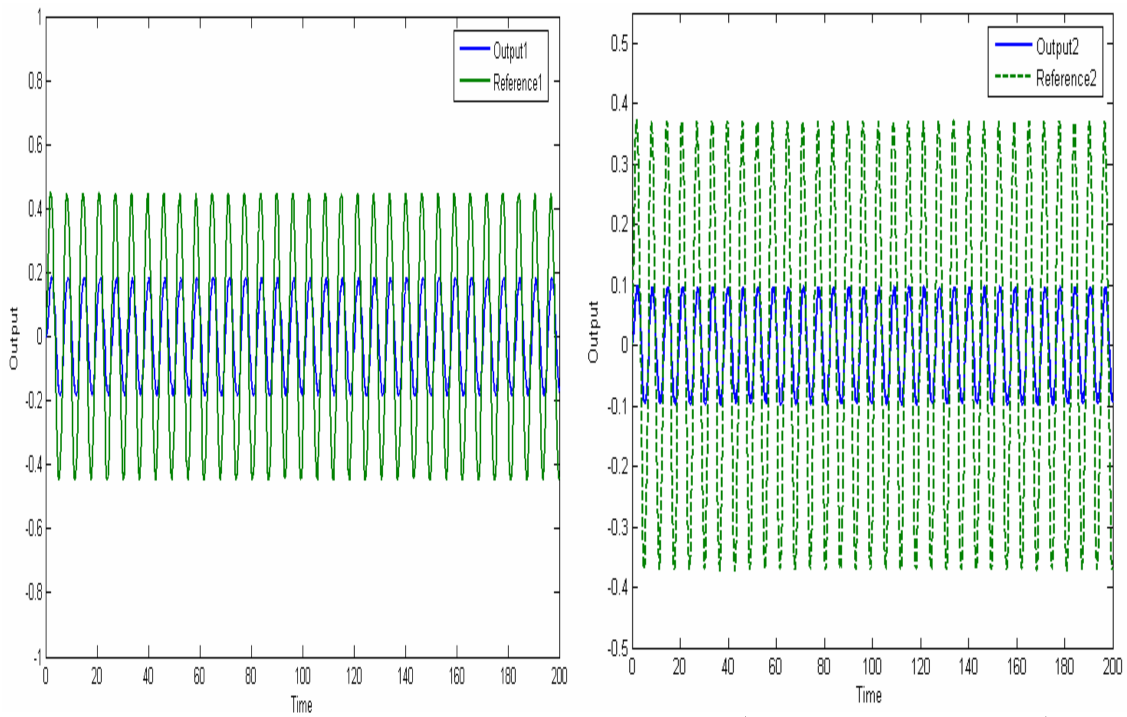


Fig. 6.13 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

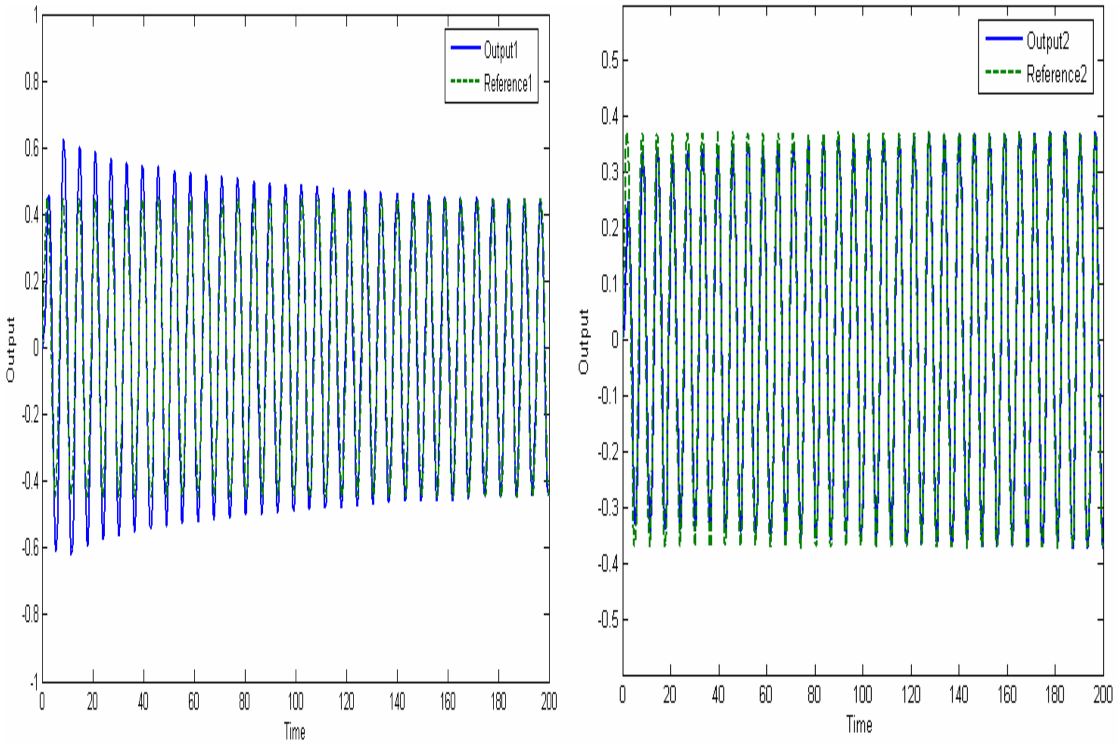


Fig. 6.14 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

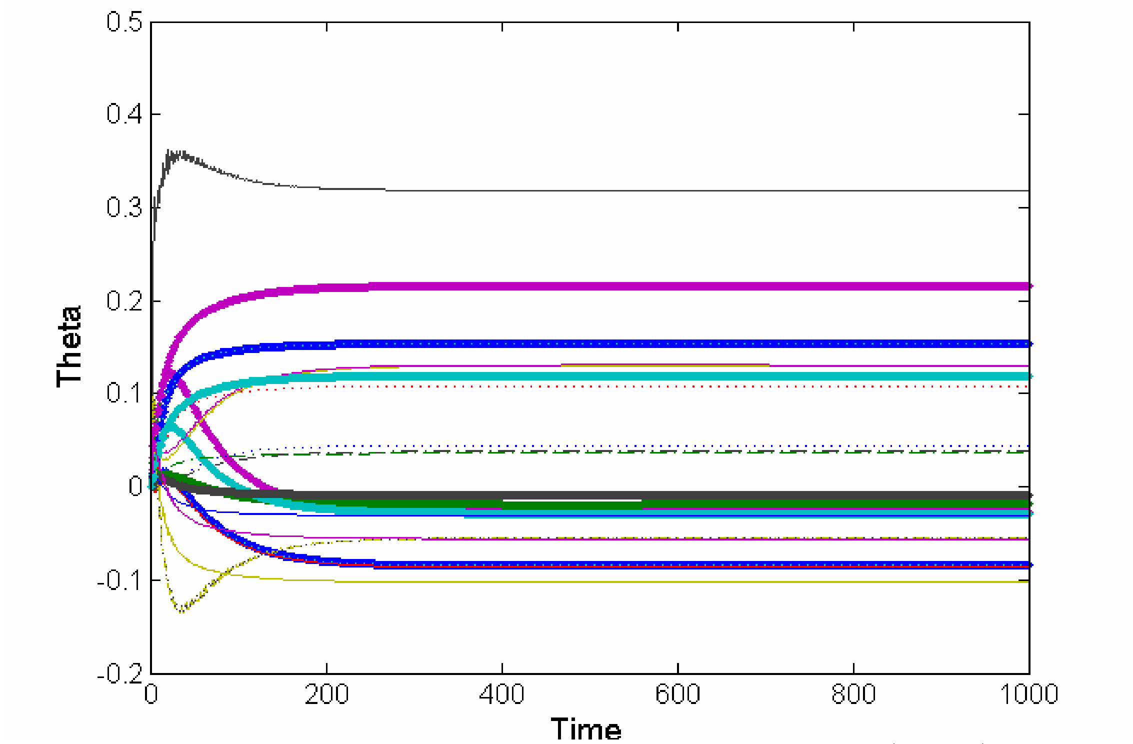


Fig. 6.15 Time Evolution of Tuneable Parameters (Theta)

Case3: for $r(t)$ = Random Signals

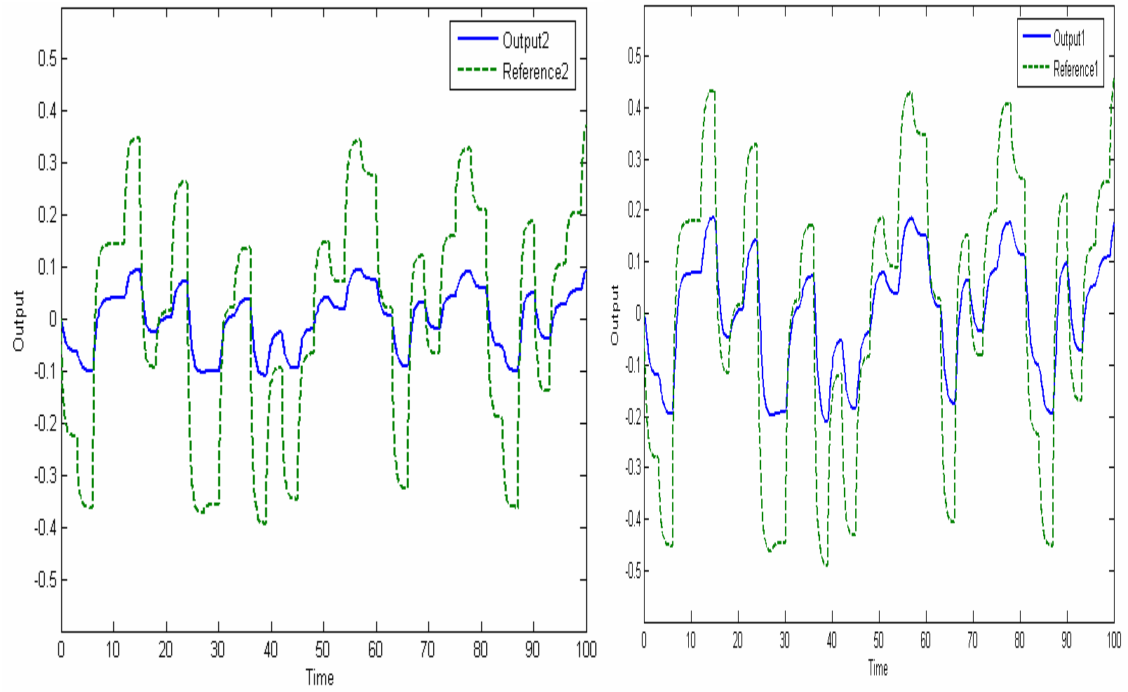


Fig. 6.16 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

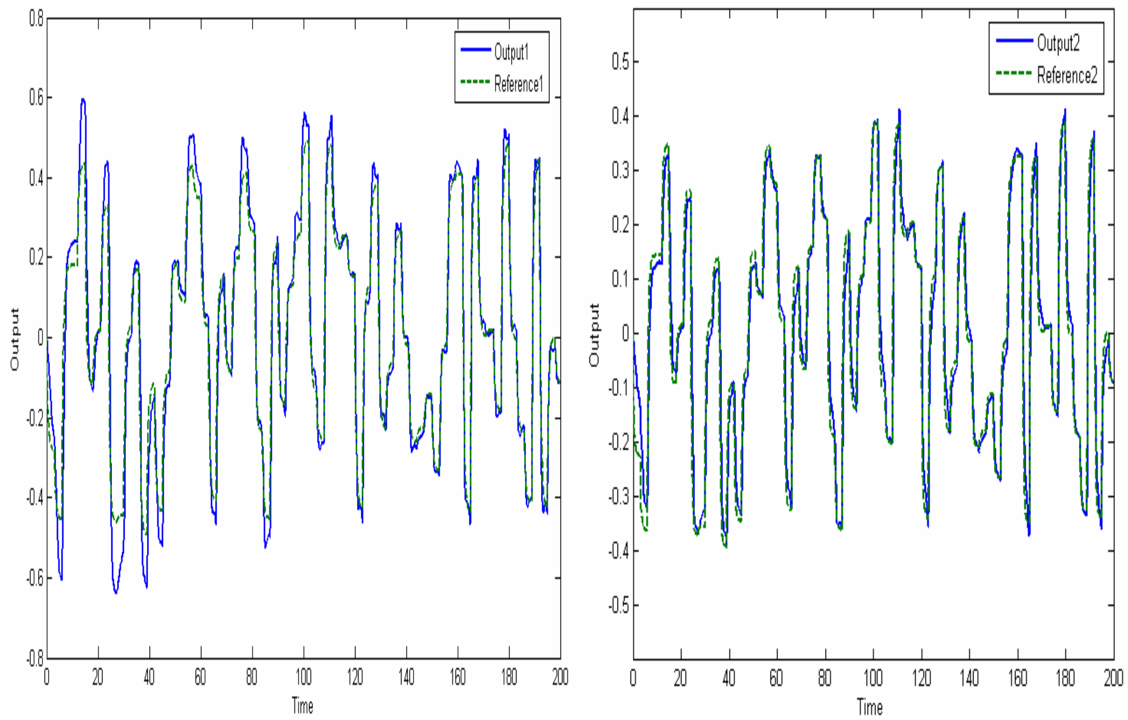


Fig. 6.17 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

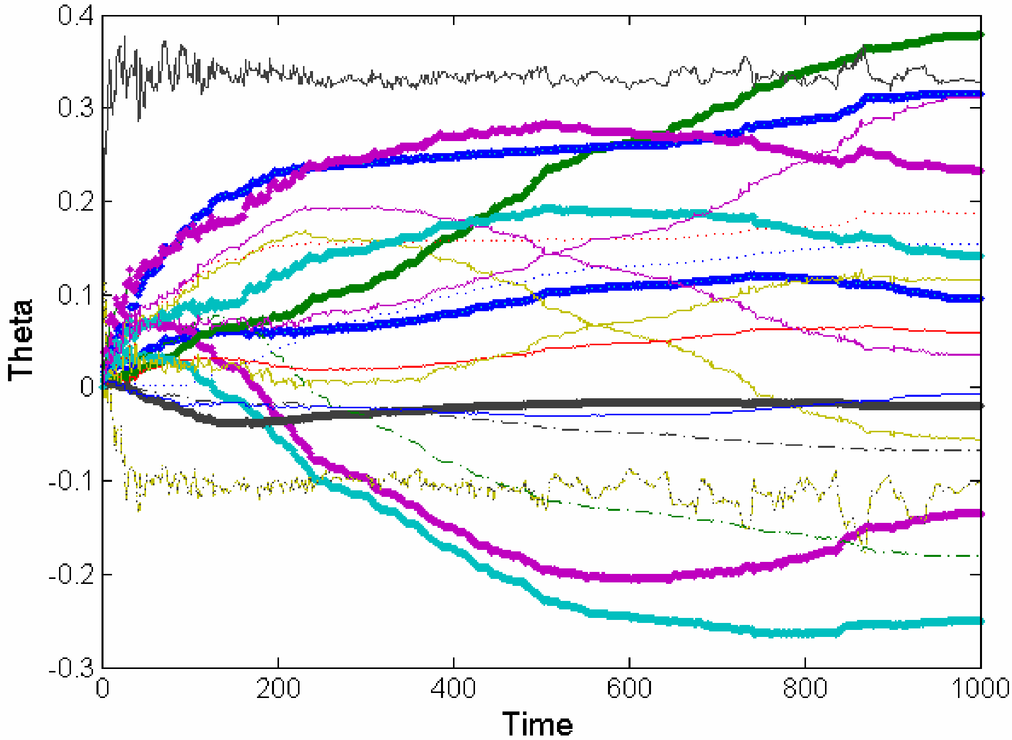


Fig. 6.18 Time Evolution of Tuneable Parameters (Theta)

6.3 Example 3:

Giving

$$A=\begin{pmatrix}1 & 0 & -3\\ 2 & -4 & 1\\ 3 & 1 & -5\end{pmatrix}, B=\begin{pmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{pmatrix}, C=\begin{pmatrix}1 & 1 & 0\\ 0 & 2 & 1\end{pmatrix},$$

we have the following transfer matrix of the plant:

$$P(s)=\begin{pmatrix}\dfrac{s^2+4s+1}{s^3+8s^2+19s+23} & \dfrac{-2s-19}{s^3+8s^2+19s+23}\\ \dfrac{2s^2+9s+7}{s^3+8s^2+19s+23} & \dfrac{s^2+5s-18}{s^3+8s^2+19s+23}\end{pmatrix}.$$

Using (2.5), we have $\mu_1=1, \mu_2=1$ and thus we set the following diagonal inverse interactor (pre-filter) using (2.8)

$$L^{-1}(s)=W(s)=\begin{pmatrix}\dfrac{1}{s+2} & 0\\ 0 & \dfrac{1}{s+2.5}\end{pmatrix}.$$

Also, we obtain the upper bound of the relative degree using (2.12) which is equal to $\mu = 1$ in order to know the dimension of the dynamic feedforward system matrices. As a result, we set the following the following A_f and B_f for the dynamic system of the feedforward controller (4.10) based on (4.4)

$$A_f = \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix}, B_f = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

We choose the following feedback gain matrix that maintains the closed-loop stability

$$K_{fb} = \begin{pmatrix} 0.1 & 0.25 \\ -0.3 & -0.15 \end{pmatrix}.$$

Then, we use the derived learning law (4.24) to adjust the tuneable parameter $\Theta(t)$ of the feedforward controller. In order to show the tracking capability of the proposed method, we have considered three different cases of the reference input whereas each case corresponds to specific reference signals. The simulation results in Fig. 6.19, Fig. 6.22 and Fig. 6.25 show the outputs $y(t)$ versus $w(t)$ for step, sine and random reference inputs using normal feedback controller without MIMO-FEL, respectively. It can be seen clearly that the tracking performance is not good for all of the three kinds of reference signals. Thus, the outputs $y(t)$ fail to track their reference signals $w(t)$. However, the simulation results in Fig. 6.20, Fig. 6.23 and Fig. 6.26 show that we have successfully tracked the same reference signals $w(t)$ using the proposed MIMO-FEL. This also verifies that the feedback controller concerns about stabilizing the system while the feedforward improves the performance of the system. Furthermore, all the components of the parameters $F(t), G(t), and\ H(t)$ which correspond to tuneable parameter of the adaptive feedforward controller $\Theta(t)$ converge to constant matrix as shown in Fig. 6.21, Fig. 6.24 and Fig. 6.27 for step, sine and random reference signals, respectively. Since we have almost obtained perfect tracking of three different cases of the reference inputs, this concludes that the learning rule successfully learns the inverse of the MIMO system.

Case1: for $r(t)$ = Step Signals

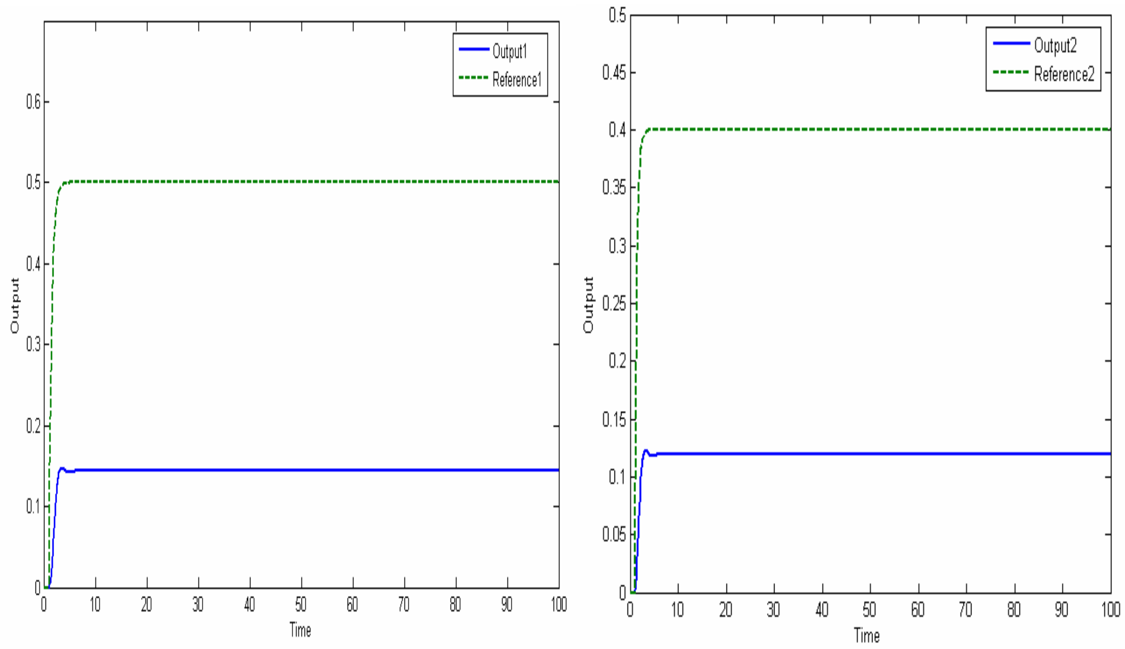


Fig. 6.19 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

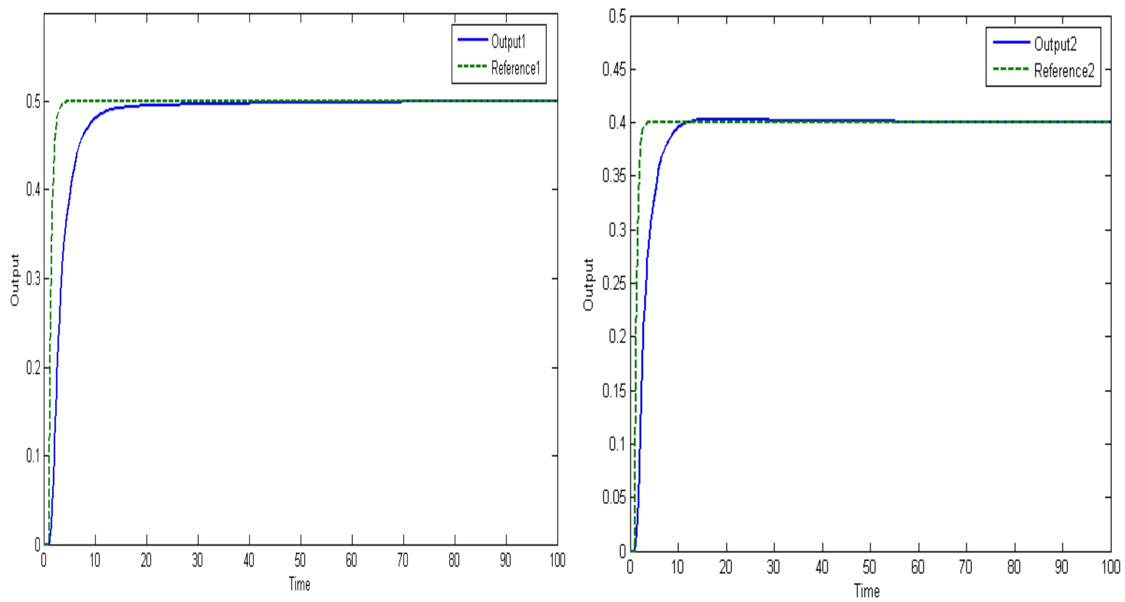


Fig. 6.20 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

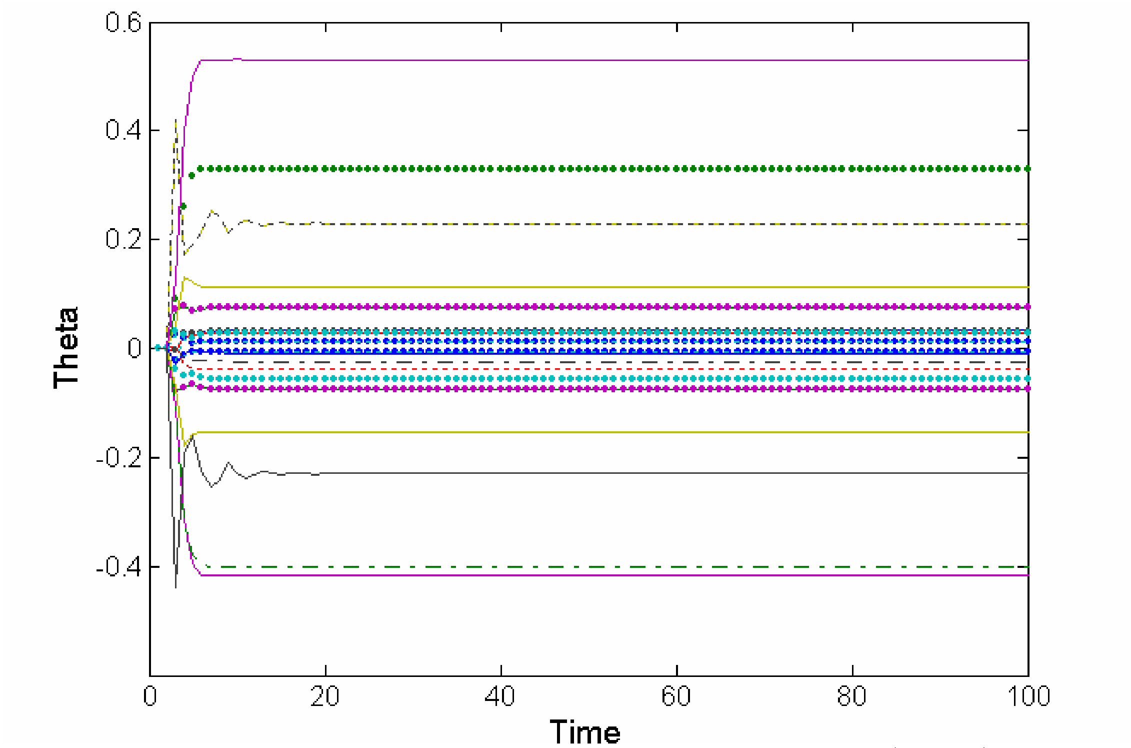


Fig. 6.21 Time Evolution of Tuneable Parameters (Theta)

Case2: for $r(t)$= Sinusoidal (sin) Signals

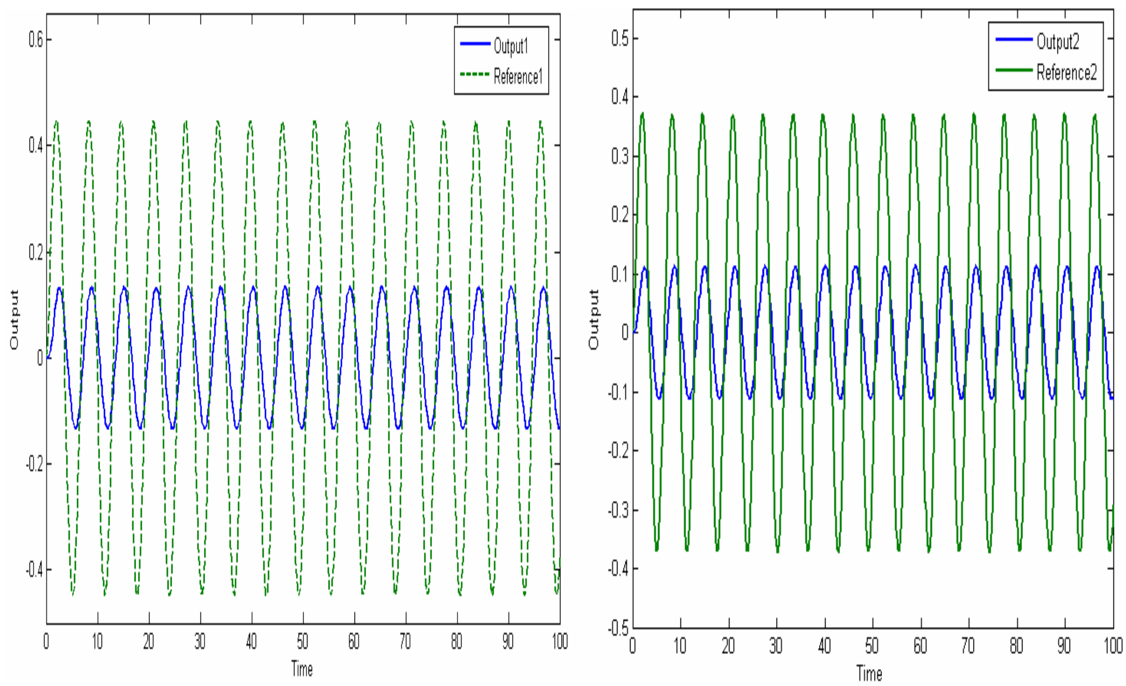


Fig. 6.22 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

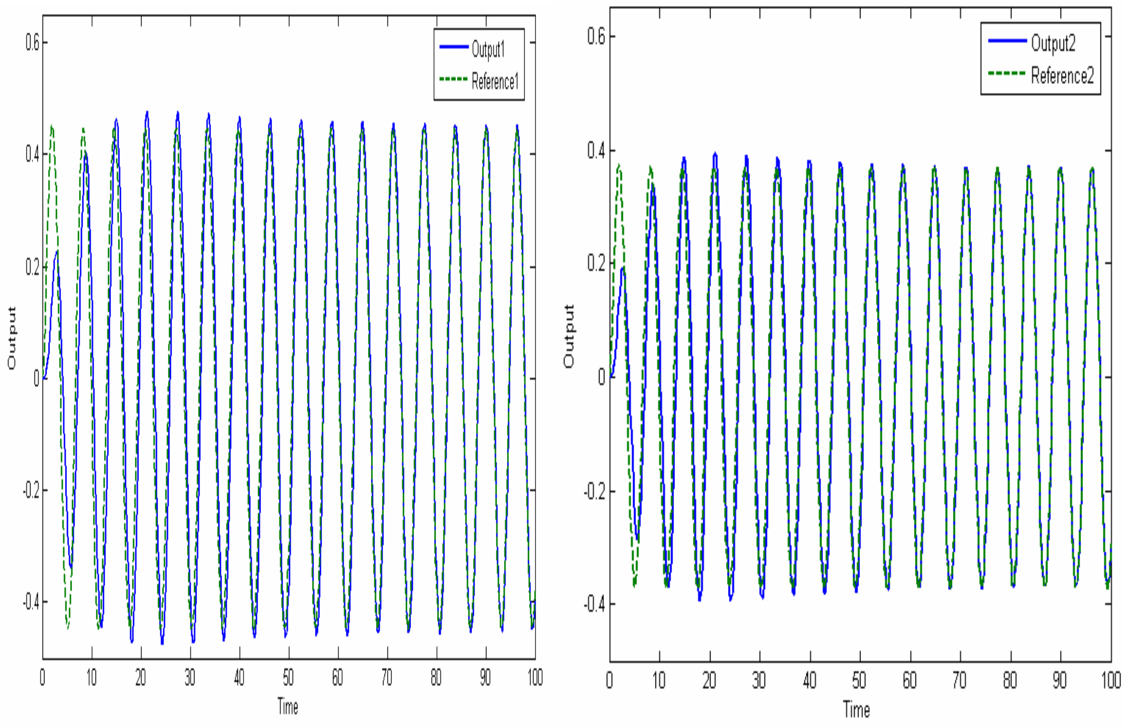


Fig. 6.23 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

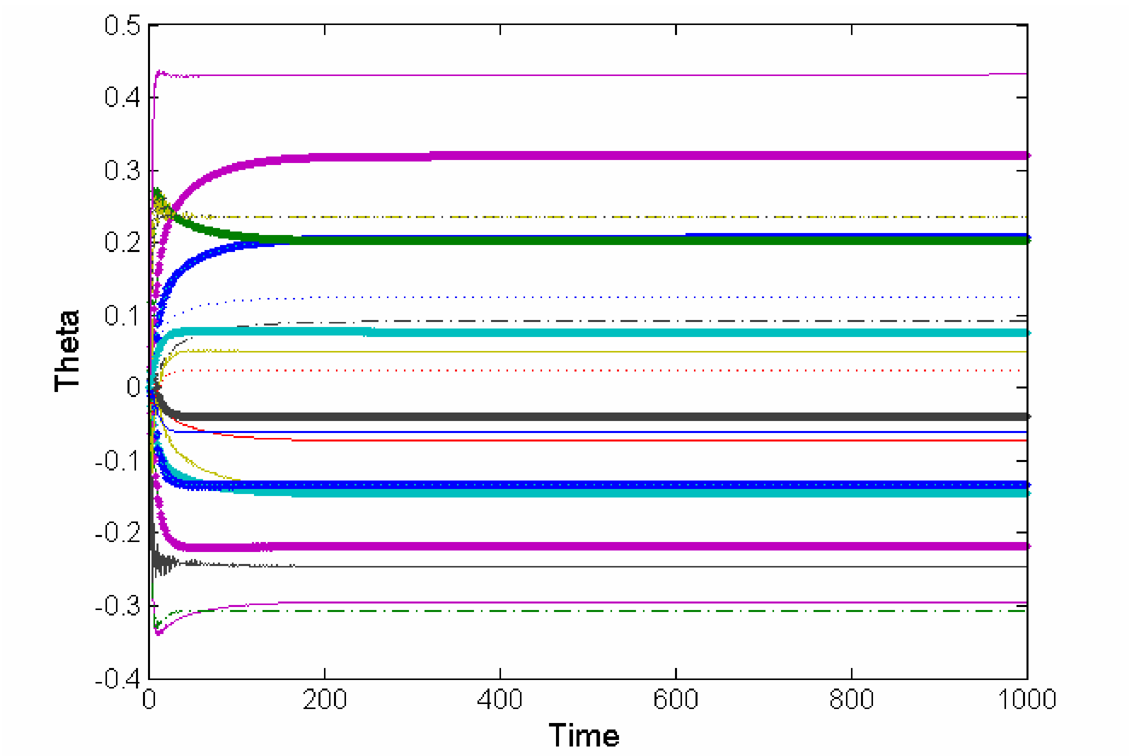


Fig. 6.24 Time Evolution of Tuneable Parameters (Theta)

Case3: For $r(t)$ = Random Signals

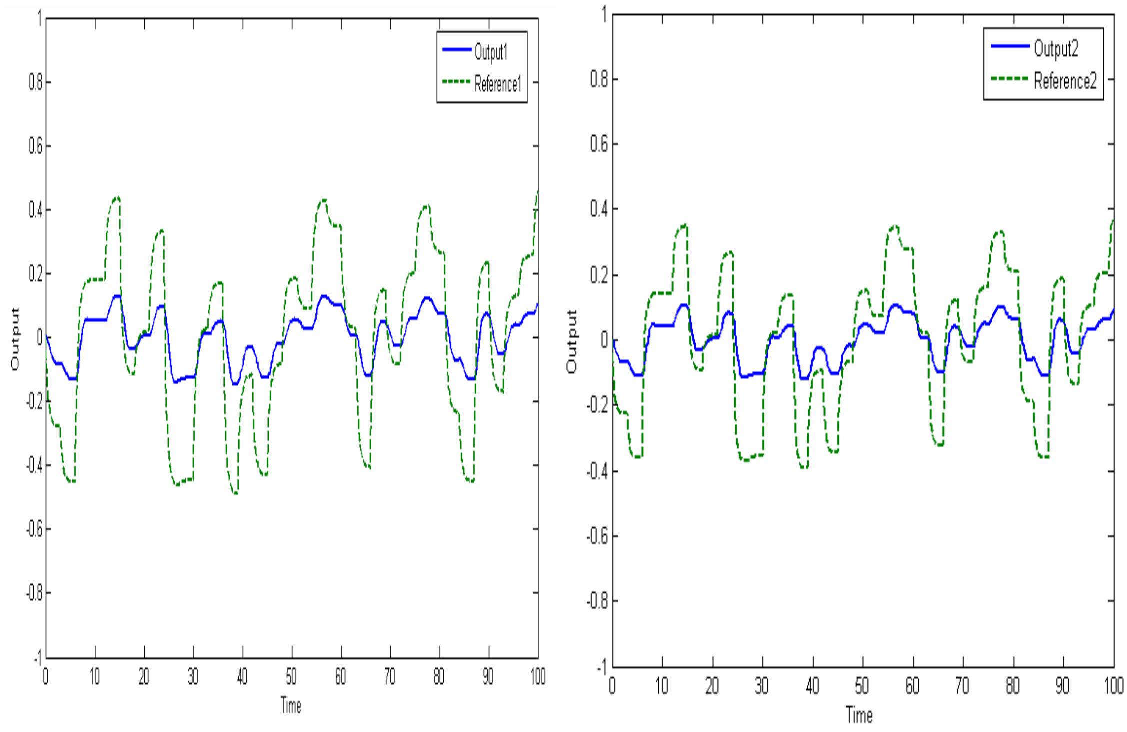


Fig. 6.25 Outputs $y(t)$ vs. References $w(t)$ (without MIMO-FEL)

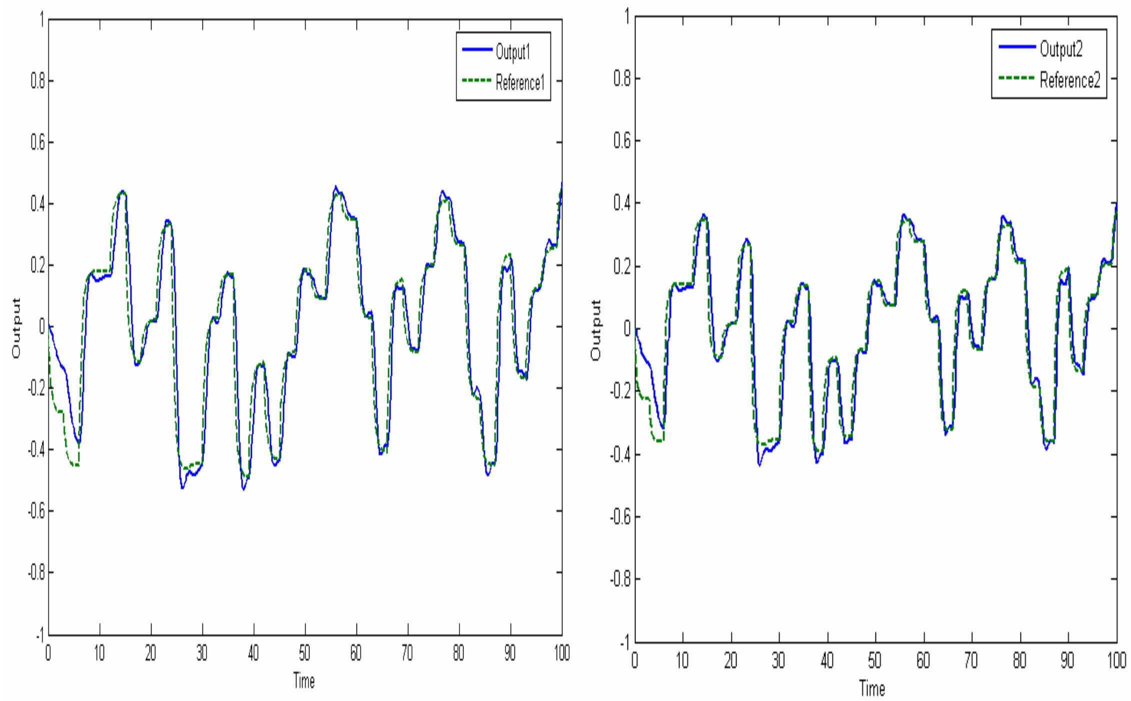


Fig. 6.26 Outputs $y(t)$ vs. References $w(t)$ (with MIMO-FEL)

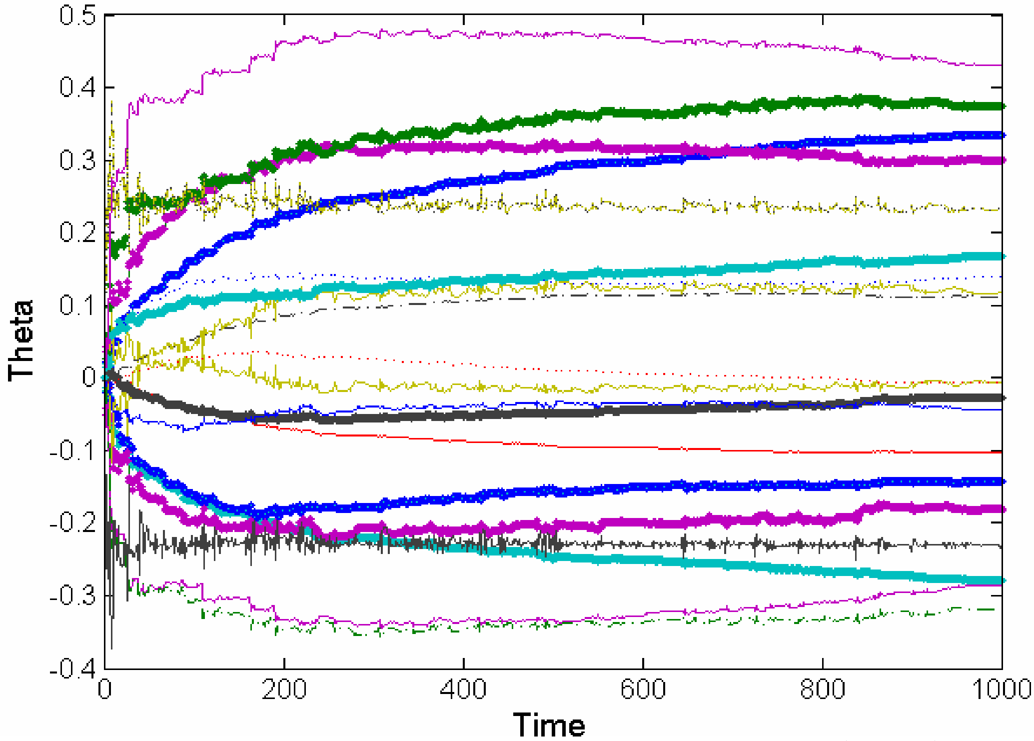
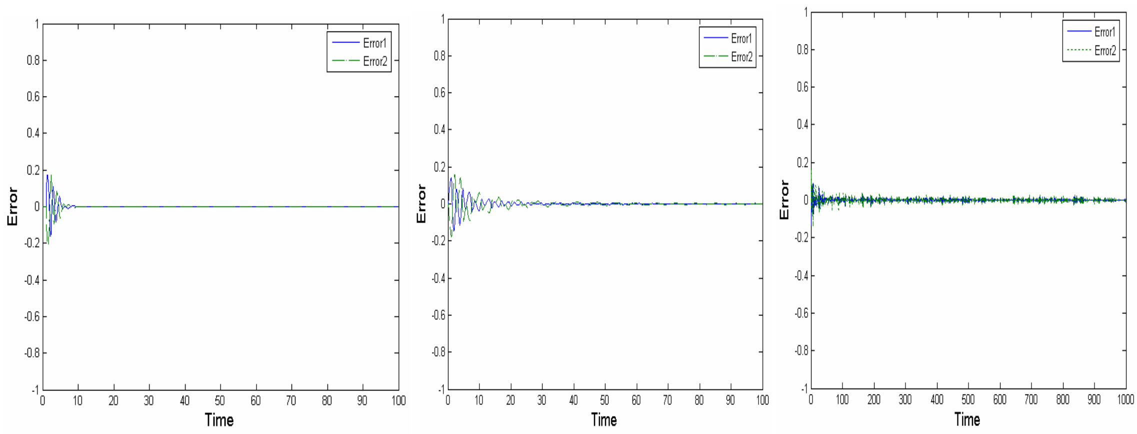


Fig. 6.27 Time Evolution of Tuneable Parameters (Theta)

Chapter 7
Analysis and Conclusion

7.1 Analysis of Results:

It can be seen from the previous simulation results that we have achieved almost perfect tracking for the plants of different orders by tracking different reference inputs as error differences shows for example 3 in Fig. 7.1 for step, sine and random reference inputs, respectively. For case1 and case2, the errors were fluctuating around zero during the transient time and then settle down exactly to zero through the steady-state time. However, in the third case, the errors are fluctuating around zero during the transient time and then tend very closely to zero. Furthermore, it has been noticed in the previous results that all components of the tuneable parameter have settled down to constant values in case of step and sine reference inputs. However, in case of random inputs, some of the components did not converge to the ideal values. The main reason of that is due to the persistence excitation (PE) condition in the theorem1 and theorem2. This means that $\xi(t)$ has not satisfied PE condition in case of random reference while it is in case of step and sine reference inputs. Thus, the MIMO-FEL learning law is continuing updates the changes in case of the random inputs such that the outputs will still track these changes. This means that the matrix components of the tuneable parameter of the feedforward controller will continue change or fluctuate around some values. This view can be seen



Case1: Step Error   Case2: Sine Error   Case3: Random Error

Fig. 7.1 Errors Record of Example 3

clearly in Fig. 6.9, Fig. 6.18 and Fig. 6.27 that some components of $\Theta(t)$ continue change or fluctuate around some values in order to keep tracking the random references. However, if the reference input is sine or step (case1 or case2), the tuneable parameter $\Theta(t)$ settle to a constant matrix. For case2 of the third example, we have obtained the following matrix:

$$\Theta_{0_{case2}} = \begin{pmatrix} 0.2327 & 0.2327 & 0.1223 & -0.1243 & 0.2071 & 0.2071 \\ -0.0935 & -0.0935 & -0.0533 & 0.0617 & -0.2140 & -0.2140 \end{pmatrix},$$

Thus, the matrix $\Theta_{0_{case2}}$ can be used in (4.6) to find the equivalent ideal feedforward controller transfer matrix (inverse model). By using the previous matrix, we can obtain the following steady-state constant matrices using (4.2):

$$F_{0_{case2}} = \begin{pmatrix} 0.2327 & 0.2327 \\ -0.0935 & -0.0935 \end{pmatrix}, \; G_{0_{case2}} = \begin{pmatrix} 0.1223 & -0.1243 \\ -0.0533 & 0.0617 \end{pmatrix},$$
$$and \;\; H_{0_{case2}} = \begin{pmatrix} 0.2071 & 0.2071 \\ -0.2140 & -0.2140 \end{pmatrix}.$$

By substituting the above matrices and the defined A_f and B_f in the example 3, the equivalent transfer matrix can be easily obtained as follows

$$\begin{aligned} Q_{case2}(s) &= (I - G_0(sI - A_f)^{-1}B_f)^{-1}(H_0 + C_0(sI - A_f)^{-1}B_f) \\ &= \begin{bmatrix} \dfrac{0.2071s^2 + 1.254s + 1.669}{s^2 + 5.184s + 6.431} & \dfrac{0.2071s^3 + 1.669s^2 + 4.411s + 3.807}{s^3 + 7.184s^2 + 16.8s + 12.86} \\ \dfrac{-0.214s^3 - 1.821s^2 - 5.036s - 4.502}{s^3 + 8.184s^2 + 21.98s + 19.29} & \dfrac{-0.214s^2 - 1.179s - 1.594}{s^2 + 5.184s + 6.431} \end{bmatrix}. \end{aligned}$$

The interpretation of this phenomenon is that once the tuneable parameter tends to the constant matrix or converges to the ideal value, the changing of MIMO-FEL learning law will stop. This means that we have obtained the inverse of the system or perfect tracking.

Furthermore, we have testified the capability of MIMO-FEL in the presence of white noises as shown in Fig. 7.2. Fig. 7.3 shows the tuneable parameter subject to the noise. This verifies the robustness of the MIMO-FEL. However, we have noticed that once we include the white noises, the computations become so heavy and after 50 sec the program suddenly stopped. We regard this reason to the computation problems, such as dividing by very small number which is very close to zero, because the performance of the system does not show any divergence as shown in Fig. 7.1. This could be one of the challenges for the practicality of the proposed

MIMO-FEL that requires further investigation.

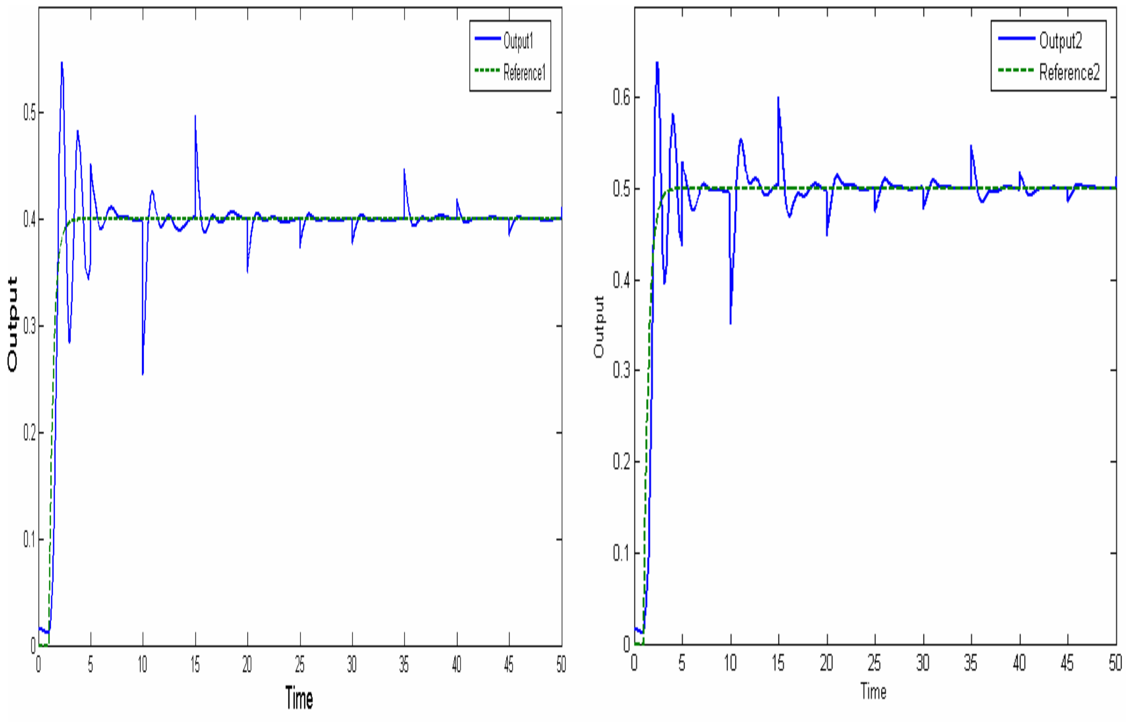


Fig. 7.2 Outputs $y(t)$ vs. References $w(t)$ Subject to White Noises

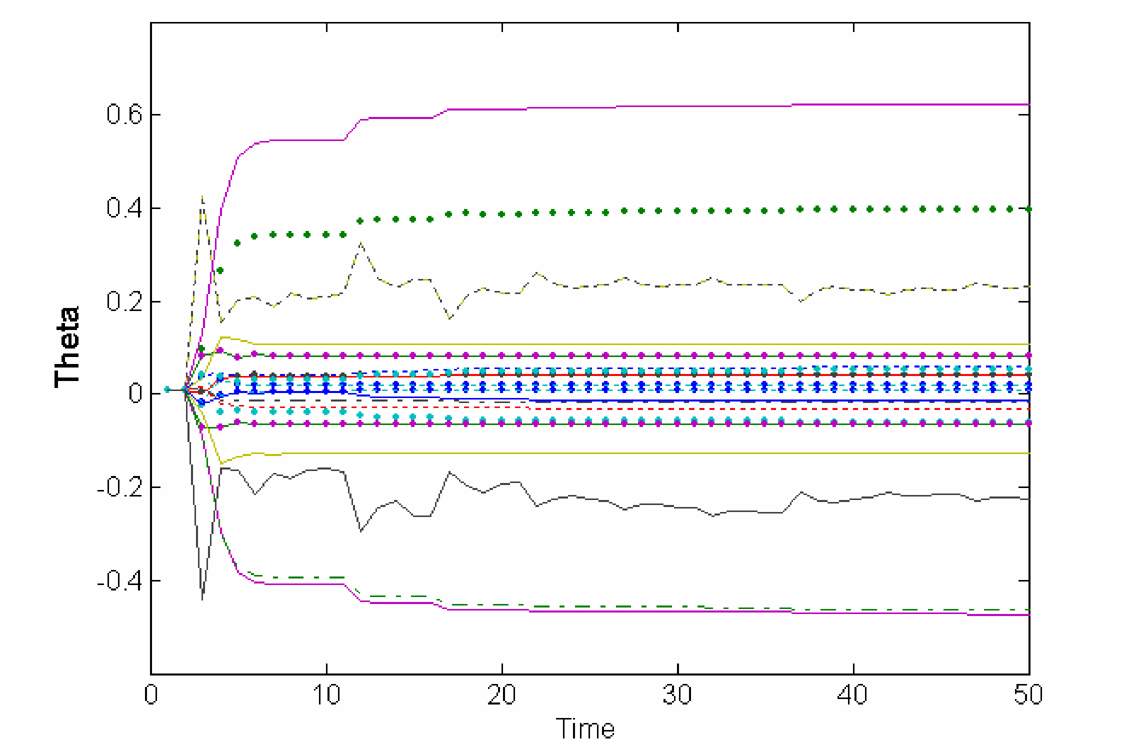


Fig. 7.3 Time Evolution of Tuneable Parameters Subject to White Noises

7.2 Drawbacks

The main drawback of the first theorem is that there is no specific criterion on selecting the feedback gain matrix. Although, the feedback guarantees sometime the closed-loop stability but it does not satisfy the SPR condition. As a result, this may lead to instability of the system once we include the feedforward controller in order to improve the performance of the system. Thus, the feedforward controller may destroy the closed-loop stability but it can not maintain the stability. Therefore, the selection of the feedback gain has to be considered seriously if we use the parameterization of the first theorem. Unfortunately, the only thing that we could do in the case of the selecting the feedback gain is trial and error. However, this drawback has been overcome by deriving the second theorem. Thus, the only thing that we should concern is to select the gain that maintains the closed-loop stability without feedforward controller. Once we use the same parameterization of second theorem for the feedforward controller, we do not have to pay attention about the stability because it is guaranteed as proved by second theorem.

The other drawback of our work is regarding to the selection of the interactor. In our work, we have assumed that we know the order and the relative degree of the plant in order to design the pre-filter and specify the dimension of the feedforward dynamic system matrices. Thus, for the sake of practicality, it requires further investigation. Moreover, there is no special criterion of selection α in the derived MIMO-FEL law but it is based on trade off which results on how fast a response is really required to track the reference input.

7.3 Future Works

For the theoretical view, the result of this work can be extended to time delay system which is similar to work done for SISO in [4] where the feedback loop of FEL system has a constant time delay [6]. Also, the comparison between FEL and other adaptive laws could be done in order to see the effectiveness of the FEL compared to other techniques. As mentioned earlier that FEL does not require extensive modelling of the system but what is the cost that we have to pay could be subject of further analysis by doing this kind of comparison. Furthermore, it would be interesting to extend FEL for linear time variant plant and nonlinear plant for SISO and MIMO systems which have not yet been done to the best of my knowledge.

On the other hand, from the practical standpoint, since the simulation results are so impressed to investigate the practicality of the proposed method due to the fast convergence, small transient time and almost perfect tracking. At first, we are looking forward to verify and validate the proposed method through prototype models such as inverted pendulum or helicopter model. Then, if we success or achieve good result using the prototype model, we would like to testify the proposed MIMO-FEL through real application if that will be possible.

7.4 Conclusion

Through this work, a theoretical treatment of how to generalize feedback error learning (FEL) for MIMO system has been studied in the framework of adaptive control. The FEL scheme is generalized to MIMO systems which are not necessarily biproper, and hence not invertible with properness. The feedforward controller is not designed on the basis of the process model. However, it is trained on-line during control using feedback error signal as a learning signal.

At first, the ideal inverse of the system is derived by means of interactor concept as presented in chapter 2. The main idea of ideal inverse of the system is assuming that a good representation of the plant exists by means of system identification and modelling. As a result, the ideal inverse of the system based on the approximated parameters has been derived using a pre-filter in order to compensate the properness of the system in chapter 2.

However, since the plant parameters are always unknown or difficult to obtain, we derive a MIMO adaptive feedforward control using simple linear parameterization. Thus, the adaptive law for feedforward controller based on generalization of FEL for MIMO system is derived using ideal and adaptive feedforward controller as presented in chapter 4.

Furthermore, the stability of the derived adaptive control law has been proved with and without the positive realness condition through first and second theorem which are stated in chapter 5. As a result, asymptotic relation between the ideal feedforward based on the interactorization concept and adaptive one based on FEL is achieved. The simulation results in chapter 6 illustrate the effectiveness of the proposed MIMO-FEL.

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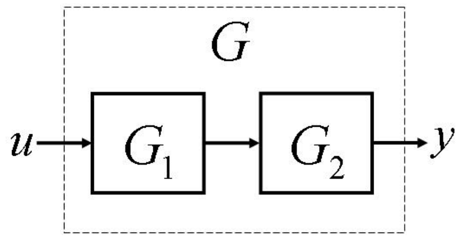
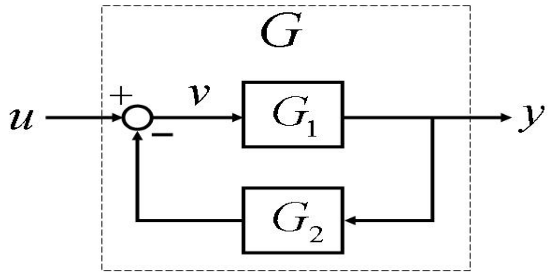
Appendices

A. MIMO System Rules



(a) Cascade System



(b) Feedback System

Fig. A Block Diagram for the Cascade and Feedback system Rule

1. Cascade rule: For the cascade (series) interconnection of G_1 and G_2 in Fig. 3(a) the overall transfer function matrix is

$$G = G_2 G_1, \tag{2.1}$$

(from right to left).

2. Feedback rule: With reference to the feedback system in Fig. 3(b), we have $v = (I + G_2 G_1)^{-1} u$ and the overall closed-loop transfer function matrix from u to y is

$$G = G_1 (I + G_2 G_1)^{-1}. \tag{2.2}$$

3. Push-through rule: For matrices of appropriate dimensions

$$G_1 (I + G_2 G_1)^{-1} = (I + G_1 G_2)^{-1} G_1. \tag{2.3}$$

B. Persistence Excitation (PE) Condition

A vector $\zeta(t)$ is persistently exciting (PE) if there exists $\lambda_1, \lambda_2, \delta > 0$ such that

$$\lambda_1 I \geq \int_{t_0}^{t_0+\delta} \zeta(\tau)\zeta(\tau)^T d\tau \geq \lambda_2 I \qquad \text{for all } t_0 \geq 0 \qquad \text{B.1}$$

as defined in [20].

C. Persistence Excitation (PE) and Exponential Stability Theorem

PE and Exponential Stability Theorem (as stated in [20])***.:***

$$\dot{\phi}(t) = -gw(t)w^T(t)\phi(t), \qquad g > 0 \qquad \textbf{\textit{C.1}}$$

Let $w: R_+ \to R^{2n}$ be piecewise continuous.
If w is PE.
Then ***(C.1)*** is globally exponentially stable.