EFFICIENT SURFACE COMPLETION USING PRINCIPAL CURVATURE AND ITS EVALUATION

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ABSTRACT

Surface completion is a technique for filling missing regions in 3D models measured by range scanners and videos. Conventionally, although missing regions were filled with the similar shape in a model, the completion process was fairly inefficient because the whole region in the model was searched for the similar shape. In this paper, the completion is efficiently performed using principal curvatures of local shape. In experiments, the effectiveness of the proposed method is successfully verified with subjective evaluation. In addition, the quantitative evaluation which has not been in the literature is newly performed.

Index Terms— Surface Completion, Principal Curvature, Energy Minimization

1. INTRODUCTION

One of the main concerns in 3D modeling of real-world objects is filling holes derived from occlusions in measurement to generate complete models. For this problem, lots of surface completion methods that fill missing regions have been proposed. These methods are classified into two categories: one is based on considering the smoothness of surface shape and the other is based on using example shapes.

The former methods [1, 2, 3, 4] can fill missing regions with smooth surface patches. Although they are effective for small holes in a 3D model, unnatural shapes may be generated when the missing parts are large and the surrounding shape is complex because the methods cannot generate a complex surface. Therefore, in order to generate complex shapes in missing regions, the methods using example shapes in other parts of the object (data region) have been intensively developed [5, 6, 7, 8]. These methods [5, 6, 7] calculate the similarity between the local surface shape around the missing regions and that in the data region and fill the missing regions by copying the most similar surface patches successively. The methods can generate complex surfaces. However, a discontinuous surface is easily generated on the seam in the completed model because the surface is fixed once it is copied. For this problem, we have proposed the method based on energy minimization using the shape similarity between the missing and data regions and successfully completed the missing regions without a discontinuous surface [8].

However, it takes much cost for the conventional methods including our previous method to complete missing regions due to two facts: (1) the similarity of shape is calculated for every surface shape that is rotated every fixed degrees around the normal axis of the surface [5, 6, 7] and (2) the whole data region is searched for the most similar shape [5, 6, 7, 8].

In this paper, to make the searching process more efficient, for (1), the angle of the rotation is uniquely determined using the directions of maximum and minimum principal curvatures of local shapes. For (2), the unneeded calculation of the similarity is omitted using principal curvatures. In experiments, the effectiveness of the proposed method is subjectively verified. In addition, the quality and efficiency of surface completion are compared by the new quantitative evaluation that has not been performed in the literature.

2. EFFICIENT SURFACE COMPLETION USING PRINCIPAL CURVATURES

Whole missing regions are optimally completed by minimizing energy after initial points and faces are generated to missing regions. Although the flow of the completion process is same as our previous method [8], the process becomes more efficient using principal curvatures in this research.

In the following, after briefly introducing the energy function for surface completion proposed in [8] in 2.1, we describe the newly proposed approach for surface alignment using directions of principal curvatures for calculating the similarity in 2.2. Next, the efficient search method using principal curvatures for minimizing the energy function is described in 2.3.

2.1. Energy function based on similarity of local shape

As illustrated in Fig. 1, first, a 3D model is divided into region Ω' including missing region Ω and data region Φ that is the rest of the object. Region Ω' is determined so that a spherical area A with constant radius whose central point is in region Ω' includes at least one of the points in region Ω. An energy function is defined as the weighed sum of SSD (Sum of Squared Distances) between the points around point $\mathbf{p}_i$ in

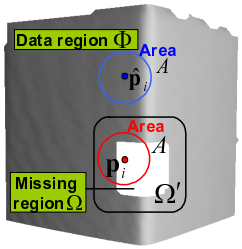


Fig. 1. Missing and data regions in a 3D model.

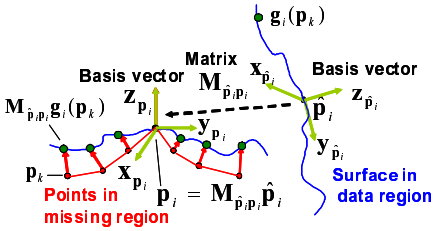


Fig. 2. Alignment of point clouds and surface.

region Ω' and surface around point $\hat{\mathbf{p}}_i$ in region Φ as follows:

$$E = \frac{\sum_{\mathbf{p}_i \in \Omega'} w_{\mathbf{p}_i} SSD(\mathbf{p}_i, \hat{\mathbf{p}}_i)}{\sum_{\mathbf{p}_i \in \Omega'} w_{\mathbf{p}_i}}, \tag{1}$$

where $w_{\mathbf{p}_i}$ is the weight for each point $\mathbf{p}_i$ [8]. $SSD(\mathbf{p}_i, \hat{\mathbf{p}}_i)$ that represents the similarity of local shape between the missing and data regions is defined as the sum of squared distances between the points in spherical area $A_{\mathbf{p}_i}$ whose central point is $\mathbf{p}_i$ and the aligned surface around $\hat{\mathbf{p}}_i$ as follows:

$$SSD(\mathbf{p}_i, \hat{\mathbf{p}}_i) = \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} \frac{\| \mathbf{p}_k - \mathbf{M}_{\hat{\mathbf{p}}_i \mathbf{p}_i} \mathbf{g}_i(\mathbf{p}_k) \|^2}{N(A_{\mathbf{p}_i})}, \tag{2}$$

where $\mathbf{M}_{\hat{\mathbf{p}}_i \mathbf{p}_i}$ denotes the transform matrix for surface alignment as shown in Fig. 2. $\mathbf{M}_{\hat{\mathbf{p}}_i \mathbf{p}_i} \mathbf{g}_i(\mathbf{p}_k)$ is a point on the aligned surface in the data region that exists in the normal direction of point $\mathbf{p}_k$ ($\in A_{\mathbf{p}_i}$). $N(A_{\mathbf{p}_i})$ is the number of points in spherical area $A_{\mathbf{p}_i}$.

2.2. Determination of transform matrix using principal curvatures of local shape

Transform matrix for surface alignment $\mathbf{M}_{\hat{\mathbf{p}}_i \mathbf{p}_i}$ consists of the 3D positions of $\mathbf{p}_i = (x_{\mathbf{p}_i}, y_{\mathbf{p}_i}, z_{\mathbf{p}_i})$ and $\hat{\mathbf{p}}_i = (x_{\hat{\mathbf{p}}_i}, y_{\hat{\mathbf{p}}_i}, z_{\hat{\mathbf{p}}_i})$, and the basis vectors for $\mathbf{p}_i$ and $\hat{\mathbf{p}}_i$: $(\mathbf{x}_{\mathbf{p}_i}, \mathbf{y}_{\mathbf{p}_i}, \mathbf{z}_{\mathbf{p}_i})$ and $(\mathbf{x}_{\hat{\mathbf{p}}_i}, \mathbf{y}_{\hat{\mathbf{p}}_i}, \mathbf{z}_{\hat{\mathbf{p}}_i})$ as follows:

$$\mathbf{M}_{\hat{\mathbf{p}}_i \mathbf{p}_i} = \begin{pmatrix} \mathbf{x}_{\mathbf{p}_i} & & -x_{\mathbf{p}_i} \\ \mathbf{y}_{\mathbf{p}_i} & & -y_{\mathbf{p}_i} \\ \mathbf{z}_{\mathbf{p}_i} & & -z_{\mathbf{p}_i} \\ 0 \quad 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{x}_{\hat{\mathbf{p}}_i} & & -x_{\hat{\mathbf{p}}_i} \\ \mathbf{y}_{\hat{\mathbf{p}}_i} & & -y_{\hat{\mathbf{p}}_i} \\ \mathbf{z}_{\hat{\mathbf{p}}_i} & & -z_{\hat{\mathbf{p}}_i} \\ 0 \quad 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

Here, the similarity SSD in an energy function largely depends on basis vectors $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ for each point. The conventional methods inefficiently calculated the similarity for every surface that is rotated every fixed degrees around the normal axis. Instead of such an inefficient search, in this research, the basis vectors are uniquely determined using the directions of normal and principal curvatures of local surface. In the following, we describe the way to determine basis vectors $(\mathbf{x}_{\mathbf{p}_i}, \mathbf{y}_{\mathbf{p}_i}, \mathbf{z}_{\mathbf{p}_i})$ for each point $\mathbf{p}_i$. First, by principal component analysis for 3D coordinates of points in spherical area $B_{\mathbf{p}_i}$ whose central point is $\mathbf{p}_i$, the coordinate system for each point is set so that x, y and z coordinates are the directions of the eigenvectors of the first, second and third eigenvalues. Next, a following quadratic surface function is fitted to points in area $B_{\mathbf{p}_i}$.

$$z(x, y) = ax^2 + by^2 + cxy + dx + ey + f, \tag{4}$$

where each parameter (a, b, c, d, e, f) is determined using the least-squares method so as to minimize the following cost Q.

$$Q = \sum_{\mathbf{p}_k \in B_{\mathbf{p}_i}} \lambda_{\mathbf{p}_k} \{z(\bar{x}_k, \bar{y}_k) - \bar{z}_k\}^2, \tag{5}$$

where $(\bar{x}_k, \bar{y}_k, \bar{z}_k)$ is the 3D coordinate of $\mathbf{p}_k$ ($\in B_{\mathbf{p}_i}$) in the coordinate system generated by principal component analysis. $\lambda_{\mathbf{p}_k}$ is the weight for point $\mathbf{p}_k$ and $\lambda_{\mathbf{p}_k} = 1$ in the data region and $\lambda_{\mathbf{p}_k} = s^{-d'}$ in the missing region (s is constant and d' is the distance from the boundary of the missing region) because points near the boundary have higher confidence than those in the center of the missing region. From the estimated quadratic surface, the directions of normal and maximum and minimum principal curvatures are set to $\mathbf{x}_{\mathbf{p}_i}$, $\mathbf{y}_{\mathbf{p}_i}$ and $\mathbf{z}_{\mathbf{p}_i}$, respectively. Basis vectors $(\mathbf{x}_{\hat{\mathbf{p}}_i}, \mathbf{y}_{\hat{\mathbf{p}}_i}, \mathbf{z}_{\hat{\mathbf{p}}_i})$ are also determined in the same way.

2.3. Efficient energy minimization by using principal curvatures

In this research, energy function E defined in Eq. (1) is minimized using a framework of greedy algorithm. In our definition of energy E, the energy for each point can be treated independently only if similar shape pairs $(\mathbf{p}_i, \hat{\mathbf{p}}_i)$ are fixed. Thus, the following two processes are repeated until the energy converges: (i) search for similar local shape keeping positions of points fixed, and (ii) parallel update of all the positions of points keeping similar shape pairs fixed. Conventionally, because the similarity SSD was calculated for all the points in the data region, the searching process was fairly inefficient in process (i). In this research, dissimilar shapes are efficiently founded and rejected using principal curvatures.

2.3.1. Efficient Search for similar local shape using principal curvatures

In the process (i), a data region is searched for similar local shape keeping the positions of all the points in the missing region fixed. In the conventional method, SSD is calculated for all the points in data region Φ and the point which gives a

minimum SSD value is determined as the most similar point $\hat{\mathbf{p}}_i$ as follows:

$$\mathbf{f}(\mathbf{p}_i) = \hat{\mathbf{p}}_i = \underset{\mathbf{p}' \in \Phi}{\operatorname{argmin}}(SSD(\mathbf{p}_i, \mathbf{p}')). \quad (6)$$

However, it takes much cost to calculate SSD for all the points. For this problem, in this research, the calculation of SSD is skipped for the point around which the local shape may not be similar to that around the target point by using the maximum and minimum principal curvatures calculated in surface fitting described in section 2.2. Concretely, the cost K that is the dissimilarity measure of curvatures is calculated for all the points in the data region as follows:

$$K_{\mathbf{p}_i\hat{\mathbf{p}}_i} = (k1_{\mathbf{p}_i} - k1_{\hat{\mathbf{p}}_i})^2 + (k2_{\mathbf{p}_i} - k2_{\hat{\mathbf{p}}_i})^2, \quad (7)$$

where maximum and minimum curvatures of target point $\mathbf{p}_i$ and corresponding point $\hat{\mathbf{p}}_i$ are set to $k1_{\mathbf{p}_i}$ and $k1_{\hat{\mathbf{p}}_i}$, $k2_{\mathbf{p}_i}$ and $k2_{\hat{\mathbf{p}}_i}$ respectively. Here, there is a high possibility that both SSD and K get low values if the surface shapes in missing and data regions are similar. Therefore, by sorting the points in the data region in ascending order according to K and calculating SSD only for the top $n\%$ points out of all the points in the data region, the calculation cost is fairly reduced.

In addition, the proposed method uses plane-symmetrical local surfaces for completion by considering that there are a lot of plane-symmetrical local shapes in an ordinaly 3D object. Concretely, by reversing the sign of a basis vector $\mathbf{x}_{\mathbf{p}_i}$ or $\mathbf{y}_{\mathbf{p}_i}$ in the matrix for surface alignment $\mathbf{M}_{\hat{\mathbf{p}}\mathbf{p}}$, SSD for the plane-symmetrical shape is also calculated.

2.3.2. Parallel Update of positions of points

In the process (ii), the positions of all the points $\mathbf{p}_i$ in the missing region are updated in parallel so as to minimize energy E defined in Eq. (1) [8]. In the following, we briefly describe the method for calculating the positions of points $\mathbf{p}_i$ for fixed similar shape pairs. First, energy E is resolved into element energy $E(\mathbf{p}_i)$ for each point in the missing region. Here, the target point to be updated is $\mathbf{p}_i$, and the position of the k-th point inside area $A_{\mathbf{p}_i}$ is expressed as $\mathbf{p}_k$ and is corresponded to $\mathbf{f}(\mathbf{p}_k)$ by Eq. (6). In this case, the point corresponding to point $\mathbf{p}_i$ is $\mathbf{g}_k(\mathbf{p}_i)$. Now, element energy $E(\mathbf{p}_i)$ can be defined in terms of $\mathbf{p}_i$, $\mathbf{g}_k(\mathbf{p}_i)$ and transform matrix $\mathbf{M}_{\mathbf{f}(\mathbf{p}_k)\mathbf{p}_k}$ for surface alignment as follows:

$$E(\mathbf{p}_i) = \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} \frac{w_{\mathbf{p}_k}}{N(A_{\mathbf{p}_i})} \parallel \mathbf{p}_i - \mathbf{M}_{\mathbf{f}(\mathbf{p}_k)\mathbf{p}_k}\mathbf{g}_k(\mathbf{p}_i) \parallel^2 . \quad (8)$$

The relationship between total energy E and element energy $E(\mathbf{p}_i)$ for each point can be written as follows:

$$E = \sum_{\mathbf{p}_i \in \Omega} E(\mathbf{p}_i) + C. \quad (9)$$

C is the energy for the points in region $\overline{\Omega} \cap \Omega'$, and is treated as a constant because positions of points and all the similar shape pairs are fixed in this region in the process (ii).

Here, it should be noted that all the corresponding points $\mathbf{M}_{\mathbf{f}(\mathbf{p}_k)\mathbf{p}_k}\mathbf{g}_k(\mathbf{p}_i)$ $(\forall \mathbf{p}_k \in A_{\mathbf{p}_i})$ exist in the direction of normal vector for point $\mathbf{p}_i$. Thus, on the assumption that normal vector $\mathbf{n}_{\mathbf{p}_i}$ does not change after updating the position of $\mathbf{p}_i$, whole energy E can be minimized by minimizing element energy $E(\mathbf{p}_i)$ independently. The position $\mathbf{p}_i$ that minimizes $E(\mathbf{p}_i)$ is calculated as follows:

$$\mathbf{p}_i = \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} w_{\mathbf{p}_k}\mathbf{M}_{\mathbf{f}(\mathbf{p}_k)\mathbf{p}_k}\mathbf{g}_k(\mathbf{p}_i) / \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} w_{\mathbf{p}_k}. \quad (10)$$

In addition, in order to efficiently complete the missing regions, a coarse-to-fine approach is also employed for energy minimization. Concretely, the radius of spherical area $A_{\mathbf{p}}$ is decreased and the density of points in a missing region is increased step-by-step.

3. EXPERIMENT

The proposed method is applied to 3 models with holes shown in Figs. 3(a), 3(d) and 5(a) and the completed results are evaluated subjectively and quantitatively. We also verify the efficiency of the proposed method. Before applying the completion process, target holes are specified manually. As initial points and faces for the missing region, the gravity point of the boundary points of the missing region and the median points between the gravity and boundary points were given and faces were generated so as to connect these points.

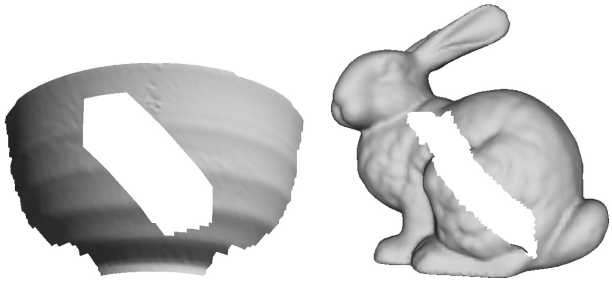
In this experiment, as the measure of the quantitative evaluation, we employ RMSD (Root Mean Squared Distance) that is defined based on the distance between points in an original model and the surface in the completed model as follows:

$$RMSD = \sqrt{\sum_{\mathbf{p}_i \in \Omega} \parallel \mathbf{p}_i - \mathbf{h}(\mathbf{p}_i) \parallel^2 / N_\Omega l^2}, \quad (11)$$

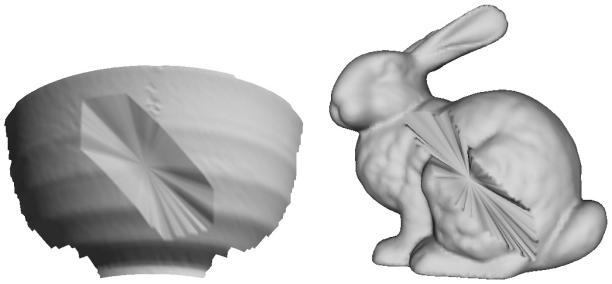
where $\mathbf{p}_i$ is the point in the original model, $\mathbf{h}(\mathbf{p}_i)$ is the point on surface in the completed model which exist in the normal direction of $\mathbf{p}_i$, N_Ω is the number of points in region Ω in the original model and l is the average length of side of mesh in the data region.

Fig. 3(b) and (e) show the models with initial points and faces. By refining these initial models with our method, the missing regions of both models are completed with plausible surfaces as illustrated in Fig. 3(c) and (f). Fig. 4 shows the changes in energy and RMSD for each model. In Fig. 4(a), the initial energy is normalized to 1. From this graph, the energy and RMSD for each model are gradually decreased by the iteration process.

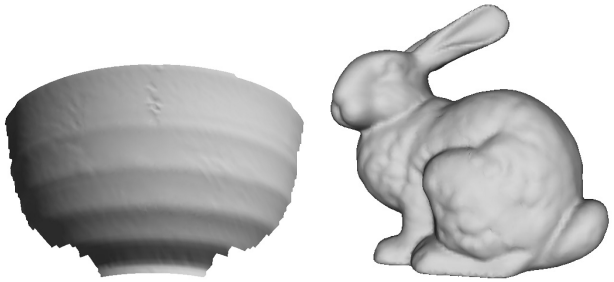
Next, to verify the effectiveness of cost reduction using principal curvatures, the processing time and the completed models are compared using PC (CPU:Xeon 3.0GHz, Memory: 8GB). Table 1 shows the processing time and RMSD with and without the proposed method. From this table, by



(a) Bowl model with a hole (d) Stanford Bunny with a hole



(b) Initial model (e) Initial model



(c) Completed model (f) Completed model

Fig. 3. Completion for models with a hole.

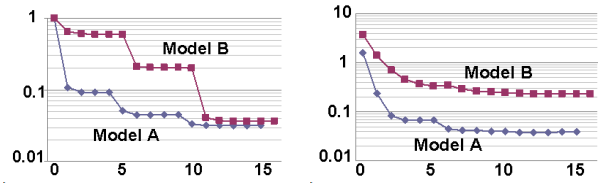
skipping the calculation of SSD, we can confirm that the computational cost is drastically reduced while keeping the quality of the completed models.

Fig. 5 shows the result of completion for a real environment. The hole on the wall of a real building and the ground in this model is generated by the street lamp. Our method also successfully works not only for synthetic holes but also for a hole in such a realistic scene.

4. CONCLUSION

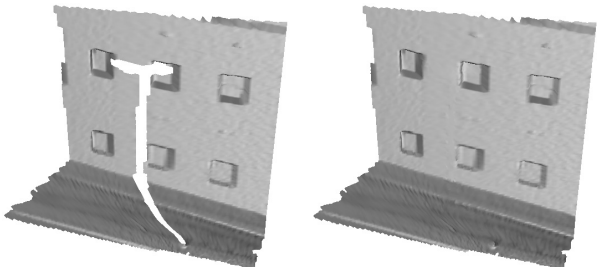
In this paper, we have presented an efficient method for surface completion using principal curvatures. In the experiment, our method have successfully and efficiently generated natural surface shapes. In future work, completion method for both shape and color will be investigated.

Acknowledgement
This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Scientific Research (A), 19200016, and Young Scientists (B), 20700162.



(a) Energy (b) RMSD

Fig. 4. Change in energy and RMSD by iteration process.



(a) Model with a hole (b) Completed model

Fig. 5. Completion for a real environment.

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Table 1. RMSD and processing time for each model when computational cost is reduced or not. SSD is calculated for the top 10% points out of all points in the proposed method.

	RMSD		Time (sec)	
Cost reduction	On	Off	On	Off
Model (I)	0.0396	0.0399	733	4072
Model (II)	0.2275	0.2472	3586	21733