Active sound absorber using adaptive signal processing

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An active absorber of sound using an adaptive algorithm is studied. To realize an active sound absorber, the incident wave on the absorber which is usually mixed with the reflected wave, must be known. In conventional methods, in order to separate the incident wave from the reflected wave using two microphones, the delay time during which the sound propagates between the two microphones must be precisely known and the two microphones must have exactly the same sensitivity. We propose two methods using adaptive processing, which do not require such preprocessing for extracting the incident wave: an off-line method using an impulse as a learning signal and an on-line method using sound intensity control. The off-line method, in which the optimum conditions are learned efficiently, yields sound absorption coefficients of more than 99% at low frequencies. In the on-line method, which do not require such learning signals as the off-line method, sound absorption coefficients of more than 93% can be realized at low frequencies.

Keywords: Active noise control, Sound absorption, Adaptive algorithm, Sound intensity, Non-reflection termination

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1. INTRODUCTION

Sound-absorptive treatments, in order to obtain high performance using conventional porous materials and resonators, require a large volume of material to absorb sound energy especially at low frequencies. The active control technique is thought to be as effective way to compensate for this deficiency in the passive method.

Previous work on this problem yielded successful results for periodic signals using the analog technique. Other work in which digital signal processing is employed allows control of the acoustic impedance using the adaptive technique.

In these methods, however, it is necessary to extract the incident wave on the absorber which is mixed with the reflected wave. This is difficult since the delay time during which the sound propagates between two microphones must be precisely known and the two microphones must have exactly the same sensitivity. In this paper, we propose two methods using adaptive processing, which do not require such preprocessing for extracting the incident wave: an off-line method using an impulse as a learning signal and an on-line method using sound intensity control. Experimental results which demonstrate the performance of the active sound absorber are shown.

2. PRINCIPLE

As shown in Fig. 1, the secondary source is located at the end of an acoustic tube and a microphone is located in front of the secondary source. Using the frequency domain equation, the output signal of the microphone is written as
\[ U(\omega) = D(\omega) + R(\omega) + S(\omega), \]  
\[ S(\omega) = C(\omega)X(\omega) = C(\omega)H(\omega)U(\omega), \]  
where \( D(\omega) \) is the direct sound, \( R(\omega) \) is the reflected sound and \( S(\omega) \) is the sound generated from the secondary source. If the transfer function between the input of the secondary source and the output of the microphone \( C(\omega) \) is known, \( S(\omega) \) is calculated using

\[ R(\omega) + S(\omega) = 0. \]

By substituting Eqs. (1), (3) to Eq. (2), the filter response \( H(\omega) \) is expressed by

\[ H(\omega) = -\frac{R(\omega)}{C(\omega)D(\omega)}. \]

In order to determine the filter response, both the direct component \((D(\omega))\) and the reflected one \((R(\omega))\) must be known. The crucial point is how these components are separated. In previous studies, a pair of microphones were used as the error sensors to separate these components. The difficulty of these approaches lies in the method of extracting the error signal, that is, the delay time during which the sound propagates between two microphones must be precisely known and the two microphones must have exactly the same sensitivity. In order to solve these practical problems, we propose two methods using adaptive processing as follows:

1. Off-line control method

In the adaptation process, an impulse is generated at the inlet of the acoustic tube as a learning signal. After the adaptation, the reflected wave is canceled by the adaptive system and only the direct wave remains.

2. On-line control method

In this method, a frequency domain algorithm based on sound intensity control is used. In this case, it is not necessary to know the delay time of the sound propagation between two microphones and the microphones need not have the same sensitivity.

3. OFF-LINE CONTROL

3.1 Adaptive Algorithm

As shown in Fig. 2, an impulse is generated at the inlet of the acoustic tube as a learning signal. First, the direct wave \( d(t) \) arrives at the microphone, followed by the reflected wave \( r(t) \). Assuming a practical situation such that we can measure only the mixed signal \( u(t) \) of the direct wave \( d(t) \) and the reflected wave \( r(t) \), it is convenient to choose the error criterion as

\[ J = \int_0^\infty u(t)^2 dt. \]

As shown in Fig. 3, it is possible to locate the

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Fig. 1: Basic diagram of active sound absorber.

Fig. 2: Concept of off-line control.

Fig. 3: Separation of direct wave and reflected wave at time \( t_0 \).
microphone so that the direct wave \( d(t) \) is separated clearly from the reflected wave \( r(t) \) at time \( t_0 \), so
\[
\begin{align*}
d(t) &= 0 & t < t_0, \\
r(t) &= 0 & t < t_0.
\end{align*}
\]
Thus, \( d(t) r(t) = 0 \) for all time and Eq. (5) can be expressed as
\[
J = \sum_{t=0}^{\infty} [d(t)^2 + r(t)^2] dt. \tag{6}
\]
It must be noted that the system cannot control the direct wave \( d(t)^2 \) because of the causality law. Accordingly, if the system is adjusted so as to minimize \( J \), only the reflected wave \( r(t)^2 \) is minimized. In the case of a discrete system, the error criterion is given by
\[
J = \sum_{i=0}^{L-1} u[i]^2, \tag{7}
\]
where \( u[i] \) is the \( i \)-th input signal of the filter and \( L \) is the length of the impulse response \( u[i] \). The adaptation algorithm which minimizes \( J \) is given by
\[
h[n+1][k] = h[n][k] - \mu \sum_{i=0}^{L-1} u[i] \sum_{j=0}^{N-1} c[j] u[i-j-k]. \tag{8}
\]
Here \( h[n][k] \) is the \( k \)-th coefficient of the adaptive FIR filter at the \( n \)-th updating, \( \mu \) is the convergence coefficient in adaptation, \( c[j] \) is the \( j \)-th coefficient of the impulse response from the secondary source to the error sensor and \( N \) is the length of the error source.

If we exchange the summations in Eq. (8), the adaptive algorithm is expressed as
\[
h[n+1][k] = h[n][k] - \mu \sum_{j=0}^{N-1} c[j] \sum_{i=0}^{L-1} u[i] u[i-j-k]. \tag{9}
\]
This equation indicates that the updating coefficients are produced by the convolution of \( c[j] \) with the auto-correlation function of \( u[i] \). As the transfer function \( u[i] \) is initially composed of regularly reflecting waves, sharp peaks and dips appear in the frequency response of the impulse response \( u[i] \), which may cause the dispersion of the updating convergence at the peak and dip frequencies. In order to avoid this, multiple error sensors are used in this study. The adaptive algorithm using multichannel error sensors is given by
\[
h[n+1][k] = h[n][k] - \mu \sum_{m=1}^{M} \sum_{i=0}^{L-1} e[m][i] \sum_{j=0}^{N-1} c[m][j] \cdot u[i-j-k], \tag{10}
\]
where \( M \) is the number of error sensors, \( e[m][i] \) is the \( i \)-th output signal of the \( m \)-th error sensor and \( c[m][j] \) is the \( j \)-th coefficient of the impulse response from the secondary source to the \( m \)-th error sensor.

3.2 Experiment

Based on the algorithm mentioned above, an active absorber was examined experimentally using an acoustic tube 2 m long with 150 mm square section. As shown in Fig. 4, four error sensors were used. One of the error sensors \( e[i] \) supplied the signal for a filter input simultaneously. A feedback-canceler was inserted in the system in order to stabilize the control.

The system was adapted to minimize the power-summation of the outputs from four error, sensors when generating an impulse (learning signal) from the primary source. Figure 5 shows the signal \( e[i] \) before and after the adaptation. After the adaptation, the reflected impulses have almost disappeared and only the direct impulse remains.

In order to evaluate the performance of this active absorber, the sound absorption coefficient was measured by the conventional standing wave method.

As shown in Fig. 6, a sound absorption coefficient

![Fig. 4 System for off-line control.](image-url)
of more than 99% was realized at almost all frequencies from 100 Hz to 1 kHz, except for 439 Hz, 782 Hz and 954 Hz. The reason for the deficiencies at these frequencies is that the magnitude of the signal $e_i$ is feeble (not zero) at these frequencies caused by the interference between the incident sound and the reflected one and hence the adaptation tends to be worse. In order to remedy these deficiencies, multiple reference sensors with a multi-channel adaptive filter system should be used.$^{4-6}$

4. **ON-LINE CONTROL**

4.1 **Adaptive Algorithm**

As algorithms for active noise control, adaptive algorithms based on sound intensity control both in time domain analysis$^7$ and in frequency domain analysis$^8,9$ have been proposed. Sound intensity control is effective for active sound absorption because it minimizes the energy flow in a certain direction. Figure 7 shows a block diagram of the intensity control system based on the two-microphone method which is widely used in sound intensity measurement.

Based on the Cross-Spectrum method, the Fourier transform of sound intensity $J(\omega)$ is given by

$$J(\omega) = \frac{\text{Im} \left( P_1^*(\omega) \cdot P_2(\omega) \right)}{\omega \rho_0 d}$$  

(11)

where $P_{1,2}(\omega)$ are the Fourier transforms of the sound pressure signal [Pa] at Mics. 1, 2, respectively, $\omega$ is the angular frequency [rad/s], $\rho_0$ is the density of air [kg/m$^3$], $d$ is the distance between the two microphones [m] and $\text{Im}(\cdot)$ indicates an imaginary part. Using the steepest descent method, the adaptation algorithm is expressed as

$$H_{n+1}(\omega) = H_n(\omega) - \mu \frac{\delta J(\omega)}{\delta H_n(\omega)},$$  

(12)

where $H_n(\omega)$ is the transfer function of the adaptive filter (n-th iteration) and $\mu$ is the convergence coefficient. The output signals of Mic. 1 and Mic. 2 are expressed as follows.

$$P_1(\omega) = X(\omega) \cdot H(\omega) \cdot C_1(\omega) + N_1(\omega)$$  

(13)

$$P_2(\omega) = X(\omega) \cdot H(\omega) \cdot C_2(\omega) + N_2(\omega)$$  

(14)
Here \( X(\omega) \) is the signal detected by the noise sensor, \( C_{1r}(\omega) \) are the transfer functions between the secondary source and Mics. 1, 2, respectively and \( N_{1r}(\omega) \) are the signals from the noise source to Mics. 1, 2, respectively. From Eqs. (11), (13) and (14), the gradient vector is expressed as

\[
\frac{\partial J(\omega)}{\partial H_s(\omega)} = \frac{-I \cdot X(\omega)}{\omega p d} (P_1(\omega) \cdot C_s(\omega)*) + P_1(\omega) \cdot C_s(\omega)* ,
\]

(15)

where \( I = \sqrt{-1} \) and * indicates conjugate.

As shown in Fig. 8, \( Q_1(\omega) \) and \( Q_2(\omega) \) are the sensitivity characteristics of Mic. 1 and Mic. 2, respectively. In this case, the output signals from Mic. 1 and Mic. 2 are expressed as

\[
P_1(\omega) = P_1(\omega) \cdot Q_1(\omega) ,
\]

(16)

\[
P_2(\omega) = P_2(\omega) \cdot Q_2(\omega) .
\]

(17)

Similarly, the transfer functions between the secondary source and the outputs of Mic. 1 and Mic. 2 are expressed as

\[
C_1'(\omega) = C_1(\omega) \cdot Q_1(\omega) ,
\]

(18)

\[
C_2'(\omega) = C_2(\omega) \cdot Q_2(\omega) .
\]

(19)

By substituting these equations into Eq. (15), we obtain the expression

\[
\frac{\partial J(\omega)}{\partial H_s(\omega)} = \frac{-I \cdot X(\omega)}{\omega p d} \left( \frac{P_1'(\omega) \cdot C_s'(\omega)*}{Q_1(\omega) \cdot Q_2(\omega)*} \right) + \frac{P_2'(\omega) \cdot C_s'(\omega)*}{Q_1(\omega) \cdot Q_2(\omega)*} .
\]

(20)

When the phase characteristics of the two microphones are identical, that is, \( \angle Q_1(\omega) = \angle Q_2(\omega) \), we obtain the following expression.

\[
Q_1(\omega) \cdot Q_2(\omega)* = Q_1(\omega) \cdot Q_2(\omega)* .
\]

(21)

In this case, Eq. (12), becomes as follows.

\[
H_{ss}(\omega) = H^*(\omega) + \frac{\mu(\omega) \cdot I \cdot X(\omega)*}{\beta} \cdot \left( \frac{P_1'(\omega) \cdot C_s'(\omega)* - P_2'(\omega) \cdot C_s'(\omega)*}{Q_1(\omega) \cdot Q_2(\omega)*} \right) ,
\]

(22)

As shown in Eq. (22), the distance between the two microphones and the characteristics of the microphones are included in the convergence coefficient \( \mu(\omega) \). That is, the filter coefficients converge regardless of the distance between the microphones and the characteristics of the microphones.

4.2 Experiment

As shown in Fig. 9, in order to examine the performance of the active absorber, an experiment was performed using the same acoustic tube as in the previous experiment. As the input signal of the system, a broad-band noise of 90–550 Hz was radiated at the inlet of the tube. The adaptation processing was performed so that the net sound intensity directed from Mic. 1 to Mic. 2 was minimized. A feedback-canceler was inserted in the system to stabilize the control. Figure 10 shows the sound absorption coefficient measured by the standing wave method. A sound absorption coefficient of more than 96% was realized in the frequency range from 100 Hz to 500 Hz. 

![Fig. 9 System for on-line control.](image-url)
Fig. 10 Sound absorption coefficients of on-line control.

5. CONCLUSIONS

Two kinds of adaptation algorithms an off-line method using an impulse learning signal and an on-line method using sound intensity control, were proposed. The experimental results obtained using the off-line method showed high absorption performance especially at low frequencies. It was found that the on-line method using a frequency domain sound intensity control algorithm is useful in practice because the filter coefficients converge regardless of the distance between the microphones and the sensitivities of the microphones.

REFERENCES


Shiro Ise was born on Mar. 13, 1961 in Tokyo, Japan. Graduated from Waseda University in 1984, and received Engineering Doctor from University of Tokyo in 1991. Research associate of Faculty of Information Science, Nara Institute of Science and Technology. Received the Encouragement Prize from the Institute of Noise Control Engineering of Japan in 1990 and the Sato Prize from the Acoustical Society of Japan in 1992.