Sound Field Reproduction by Wavefront Synthesis Using Directly Aligned Multi Point Control

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ABSTRACT
In this paper, we present a comparative study on directly aligned multi point controlled wavefront synthesis (DMCWS) and wave field synthesis (WFS) for the realization of a high-accuracy sound reproduction system, and the amplitude and phase characteristics of the wavefronts generated by DMCWS and WFS are assessed on the computer simulations and measurement in actual environments. First, the results of computer simulations revealed that the wavefront in DMCWS has wide applicability in both the spatial and frequency domains with small amplitude and phase errors, particularly above the spatial aliasing frequency in WFS. Next, we developed wavefront measurement system and measured a DMCWS wavefront with this system and proposed algorithm. The results of measurements clarified the effect of a reflected wave and the frequency characteristics of a loudspeaker. Also, DMCWS has wide applicability in frequency domains in actual environments. From these findings, we concluded the advantageousness of DMCWS compared with WFS.

1. INTRODUCTION
In recent years, there has been increasing research interest in wavefront synthesis, which enables multiple sound sources to create a sound field that is identical to any original sound field. It is expected to provide a wider effective listening area than the current 5.1 system or a surround system with many channels, which means that the listener can perceive the same sound image regardless of the listening position.

Wavefront synthesis techniques can be classified into various types; typical methods are wave field synthesis (WFS) [1, 2, 3] and directly-aligned multi-point controlled wavefront synthesis (DMCWS) [4, 5]. Although the theory of WFS has been well studied, the optimal control-point geometry and the behavior of the secondary wavefront within and above the frequency bandlimit in DMCWS have not been investigated so far. Therefore, in this paper, we describe the implementation of DMCWS and evaluate its effectiveness through the comparison with WFS. Since the wavefront has the frequency characteristics of actual audio applications, we determine the spatial spectrum characteristics of DMCWS in an actual environment. Hence, we derive the spatial spectrum characteristics from the impulse responses at each observation point and measure the DMCWS wavefront using a wavefront measurement system [6] in an actual environment.

The rest of this paper is organized as follows. In Section 2, the principles of WFS and DMCWS are explained. In Section 3, the quantitative comparison of DMCWS with WFS is described in relation to their numerically calculated wavefronts. In Sections 4, wavefront synthesis and measurement experiments in an actual environment are described. Following a discussion on the results of the experiments, we present our conclusions in Section 5.

2. THEORY

2.1. WFS
In this section, WFS and MCWS (DMCWS) are described theoretically and the equations used for numerical calculations are derived in detail. The geometric configuration and parameters in WFS are depicted in Fig. 1, where $S_p(\omega)$ and $S_n(\omega)$ denote the spectra of the primary and the nth secondary sources, respectively, on the
Fig. 1: Configuration of WFS.

The spectrum of the $n$th secondary source, which synthesizes the primary spherical wavefront, is expressed as [7, 8]

$$S_{S_{n}}(\omega) = \sqrt{\frac{k}{2\pi}}C(\rho, \theta) \frac{\exp(-jk\rho_{pn})}{\sqrt{\rho_{pn}}} \Delta x S_{p}(\omega) G(\rho_{pn}, \omega) \cos \theta_{pn},$$

(1)

where $j$ is the imaginary unit, $k$ is the wavenumber $(\omega/c)$, $c$ is the sound velocity, $\omega$ denotes the angular frequency, $\Delta x$ is the interelement interval among the secondary sources, $\rho_{pn}$ is the distance between the primary source and the $n$th secondary source, and $\theta_{pn}$ is the angle between the $y$-axis and the line connecting the $n$th secondary and primary sources. $G(\rho_{pn}, \omega)$ is a distance-independent directivity function defined only under far-field conditions. $C(\rho, \theta)$ is a function that compensates the level of mismatch due to the stationary phase approximation along the $x$-direction [9], which is a function of only the reference listening distance $y_{R}$ [10] and is given as

$$C(\rho, \theta) = \sqrt{\frac{\rho}{\rho - y_{R}}}.$$  

(2)

Outside this line, the level of the sound field is expressed as

$$A_{S_{S}}(y) = \sqrt{\frac{|y|}{|y|}} \sqrt{\frac{|y| + |y|}{|y| + |y|}} \frac{1}{|y - y_{R}|}.$$  

(3)

2.2. DMCWS

The geometric parameters of MCWS are shown in Fig. 2. MCWS controls the spatial spectra at the control points, which are randomly located on the $x$-$y$ horizontal plane in front of the secondary sources, and generates the desired wavefront. In MCWS, there is one typical case in which each control point is located on the control line parallel to the $x$-axis and intersecting the $y$-axis at $y_{C}$ (its geometric parameters are depicted in Fig. 3 [9]). In this case, the wavefront synthesis method is called DMCWS (Directly-aligned MCWS), named after its control-point geometry. Here, $S_{C_{m}}(\omega)$ denotes the secondary wavefront spectrum at the $m$th control-point position. Also, $\theta_{cm}$ is the angle between the $y$-axis and the line connecting the $m$th control point and the primary source, $r_{cm}$ is the spatial distance between the $m$th control point and the $n$th secondary source, $r_{S_{mn}}$ is the spatial distance between the $m$th control point and the $n$th secondary source, $N$ is the number of secondary sources, and $M$ is the number of control points.

Here we derive the spectrum of the secondary source $S_{S_{n}}(\omega)$, which synthesizes the primary spherical wavefront. The transfer function between the $n$th secondary source and the $m$th control point, $H_{nm}(\omega)$, is written as

$$H_{nm}(\omega) = G(\rho_{mn}, \omega) \frac{\exp(-jk\rho_{mn})}{r_{S_{mn}}}.$$  

(4)
where $G(\theta, \omega)$ is the directivity characteristic of the secondary sources. From Eq. (4), we define the transfer function matrix

$$H(\omega) = \begin{bmatrix}
H_{0,0}(\omega) & H_{1,0}(\omega) & \cdots & H_{N-1,0}(\omega) \\
H_{0,1}(\omega) & H_{1,1}(\omega) & \cdots & H_{N-1,1}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
H_{0,N-1}(\omega) & H_{1,N-1}(\omega) & \cdots & H_{N-1,N-1}(\omega)
\end{bmatrix}.$$  

We write the secondary wavefront spectrum vector at the $m$th control-point position as

$$S_C(\omega) = H(\omega)S_S(\omega),$$  

where

$$S_C(\omega) = [S_{C0}(\omega), S_{C1}(\omega), \cdots, S_{CM-1}(\omega)]^T,$$

$$S_S(\omega) = [S_{S0}(\omega), S_{S1}(\omega), \cdots, S_{SN-1}(\omega)]^T,$$

and $^T$ denotes the transpose of the vector/matrix. If the primary wavefront spectrum is equal to the secondary wavefront spectrum at the control-point position, Eq. (6) can be transformed into

$$S_C(\omega) = P(\omega)S_P(\omega),$$

where

$$P(\omega) = \begin{bmatrix}
\frac{1}{r_{C0}}, & \frac{1}{r_{C1}}, & \cdots & \frac{1}{r_{CM-1}}
\end{bmatrix}^T.$$  

From Eqs. (6) and (9) and the generalized inverse matrix of $H(\omega)$, $H^+(\omega)$, we obtain the secondary source spectrum vector in the following form:

$$S_S(\omega) = H^+(\omega)P(\omega)S_P(\omega).$$  

3. COMPUTER-SIMULATION BASED COMPARISON OF DMCWS AND WFS

3.1. Method of calculating secondary wavefront

In this section, we compare DMCWS and WFS through computer-simulation-based experiments in terms of the amplitude, phase, and attenuation of the synthesized secondary wavefront spectrum.

The secondary source and observation point geometric parameters are shown in Fig. 4. Equation (12) defines the spectrum of the secondary wavefront at the observation point, given as

$$S_O(\omega) = \sum_{n=1}^{N} S_S(\omega)G(\theta_{on}, \omega)\frac{\exp(-jkr_{on})}{r_{on}}.$$  

The secondary sources are circular vibration planes on an infinite baffle whose directional characteristic is

$$G(\theta, \omega) = \frac{2J_1(kb \sin \theta)}{kbsin\theta},$$

where $J_1(\cdot)$ is the bessel function of the first kind and $b$ is the diaphragm radius of each circular vibration plane.

3.2. Calculation conditions

The conditions of the wavefront calculation are shown in Table 1. The diaphragm radius $b$ and secondary source distance $\Delta x$ resemble those of the Soundevice SD-0.6 loudspeaker shown in Fig. 5. The evaluated wavefront band frequencies below 1600 Hz are major cues for sound source localization [11]. The geometric parameters in the wavefront calculation are illustrated in Fig. 6.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>20°C</td>
</tr>
<tr>
<td>Evaluated wavefront band frequencies</td>
<td>20-1600 Hz</td>
</tr>
<tr>
<td>Spatial aliasing frequency</td>
<td>(10 Hz interval)</td>
</tr>
<tr>
<td>Primary source geometry $(x_p, y_p)$</td>
<td>(1.2, -0.1) m</td>
</tr>
<tr>
<td>Secondary source and Control point interval $\Delta x$</td>
<td>0.12 m</td>
</tr>
<tr>
<td>Diaphragm radius $b$</td>
<td>0.05 m</td>
</tr>
<tr>
<td>Number of secondary sources $N$ and Control points $M$</td>
<td>16</td>
</tr>
<tr>
<td>Control line y-coordinate $y_c$</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Reference listening distance $y$-coordinate $y$</td>
<td>0.6 m</td>
</tr>
</tbody>
</table>

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Fig. 5: Soundevice SD-0.6 loudspeaker, whose diaphragm radius and source distance were assumed in the experiment.

![Diagram of a loudspeaker with primary and secondary sources](image)

Fig. 6: Geometric parameters in wavefront calculation.

3.3. Evaluation criteria of secondary wavefront

The evaluation criteria $E_A(i,j)$ and $E_P(i,j)$ used to evaluate the complex amplitude and phase errors of the secondary wavefront are respectively defined as

$$E_A(i,j) = \frac{1}{2} \sum_{\omega} \left( |PWF(i,j,\omega)| - |WF(i,j,\omega)| \right)^2,$$

and

$$E_P(i,j) = \frac{1}{N} \sum_{\omega} \frac{1}{\pi} \arctan \left( \frac{PO(WF(i,j,\omega))}{PO(PWF(i,j,\omega))} \right),$$

where $PWF(i,j,\omega)$ denotes the primary wavefront radiated by primary sources, $WF(i,j,\omega)$ denotes the secondary wavefront synthesized by DMCWS or WFS and $PO(\cdot)$ denotes the phase function given by

$$PO(x) = \frac{x}{|x|},$$

where $x$ is a complex-valued variable.

Fig. 7: Calculated amplitude error $E_A$ for WFS.

![Graph showing calculated amplitude error $E_A$ for WFS](image)

Fig. 8: Calculated amplitude error $E_A$ for DMCWS.

![Graph showing calculated amplitude error $E_A$ for DMCWS](image)

3.4. Calculation results

Figures 7 and 8 show the calculated values of $E_A$ for WFS and DMCWS, respectively. The values of $E_A$ are given on the contour lines, and the intervals between the contour lines are 0.5 dB in Fig. 7 and 2.0 dB in Fig. 8. As can be seen in Fig. 7, the amplitude error of WFS is serious because the evaluation frequency band is above the spatial aliasing frequency (1417 Hz). In contrast, in Fig. 8, the amplitude error of DMCWS is 6–7 dB, which is small in comparison with that of WFS, and is generally smallest in the vicinity of the control points. The amplitude error difference between the Figs. 7 and 8 indicates 13–60 dB around the control points.

Figures 9 and 10 show the calculated values of $E_P$ for WFS and DMCWS, respectively. The values of $E_P$ are given on the contour lines, and the intervals between the contour lines are 0.05 dB in Fig. 9 and 2.0 dB in Fig. 10. According to Fig. 9, there is a significant phase error $E_P$ similar to the amplitude error shown in Fig. 7, in the case of WFS. In contrast, there is an extremely small phase error in DMCWS, as shown in Fig. 10.

Figure 10 shows that in the wavefront of DMCWS, the amplitude error is large, but the phase error is small, and consequently the wavefront amplitude error is dominant in DMCWS. Therefore, we next calculate the attenuation to examine the characteristic of the wavefront amplitude error.
4. EVALUATION IN ACTUAL ENVIRONMENT

4.1. Spatial spectrum characteristics obtained from impulse responses

In this section, we propose a wavefront measurement method using the spatial spectrum characteristics obtained from the impulse response at each observation point.

The wavefront spectrum characteristics (Eq. (12)) at the observation point $S_0(\omega)$ are expressed below in vector form:

$$ S_0(\omega) = Q^T(\omega)S_p(\omega) $$

$$ = Q^T(\omega)H^+(\omega)P(\omega)S_p(\omega) $$

$$ = S_{arr}(\omega)S_p(\omega), \tag{17} $$

where

$$ Q(\omega) = [Q_0(\omega), Q_1(\omega), Q_2(\omega), ..., Q_n(\omega)]^T. \tag{18} $$

$$ Q_n(\omega) = G_n(\theta, \omega)\exp(-jk \omega r_0). \tag{19} $$

$Q_n(\omega)$ is the radiation characteristic of the nth sound source at the azimuth angle $\theta$ for angular frequency $\omega$, and the loudspeaker array characteristic $S_{arr}(\omega)$ is assumed to be the radiation characteristic of a single sound source.

Figure 11 shows observation area arranged in a reticular pattern of observation point. In Fig. 11, $S_{arr}(\omega)$ is the set of spectrum characteristic of synthesized secondary wavefront at each observation point in matrix form:

$$ S_{arr}(\omega) = \begin{bmatrix} S_{arr}(0,0)(\omega) & S_{arr}(0,1)(\omega) & \cdots & S_{arr}(0,J-1)(\omega) \\ S_{arr}(1,0)(\omega) & S_{arr}(1,1)(\omega) & \cdots & S_{arr}(1,J-1)(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{arr}(I-1,0)(\omega) & S_{arr}(I-1,1)(\omega) & \cdots & S_{arr}(I-1,J-1)(\omega) \end{bmatrix} \tag{20} $$

$I$ and $J$ is the total number of spatial sampling index of $i$ and $j$, respectively.

From Eq. (20), we obtain the temporal wavefront using inverse fourier transform;

$$ s_{arr}(t,\omega) = \mathcal{F}^{-1}[S_{arr}(\omega)]e^{i\omega t} $$

$$ = \begin{bmatrix} s_{arr}(0,0)(t,\omega) & s_{arr}(0,1)(t,\omega) & \cdots & s_{arr}(0,J-1)(t,\omega) \\ s_{arr}(1,0)(t,\omega) & s_{arr}(1,1)(t,\omega) & \cdots & s_{arr}(1,J-1)(t,\omega) \\ \vdots & \vdots & \ddots & \vdots \\ s_{arr}(I-1,0)(t) & s_{arr}(I-1,1)(t) & \cdots & s_{arr}(I-1,J-1)(t) \end{bmatrix} \tag{21} $$

where “•” denotes the quantity survey for matrix element.

In this paper, we estimate $S_{arr}(\omega)$ using the M-sequence method [12] to measure the acoustic impulse response.
4.2 Wavefront measurement system

Figure 12 shows the wavefront measurement system for visualization of the wavefront obtained by Eq. (20).

The measuring microphones are placed on the electric two axes actuator so that the microphones can move around the horizontal plane in front of the loudspeaker array and measure the spectrum characteristics $S_{arr,i,j}(\omega)$. In addition, we use a linear microphone array to save the amount of time spent for measurement. The interval of the microphones is 0.48 m and the total number of microphones is 4, i.e., microphone array width is 1.44 m, and the microphones are audio-technica ATM14a omnidirectional microphones. The electric actuator has 0.96 m range of movement on each axis. As a result, the width (x-axis) and height (y-axis) of the observation area are 2.4 m and 0.96 m, respectively.

Figure 13 shows the procedure used to construct resultant all wavefronts of all the observation areas from measured wavefront at each observation area. As shown in Fig. 13 (b), can be obtained as an overlap of each wavefront measured from adjacent microphones, shown in Fig. 13 (a).

4.3 Wavefront measurement conditions

Table 2 shows the wavefront measurement conditions. The control line y-coordinate $y_C$ is set to 0.6 m.

$$f_{alias} = \frac{c}{2\Delta x} = \frac{340}{2 \times 0.12} = 1417 \text{ [Hz]}, \quad (22)$$

where $\Delta x$ denotes the distance from the loudspeaker and $c$ denotes the sound velocity.

4.4 Results of wavefront measurement

Figures 14 and 15 show the calculated and measured wavefronts obtained using DMCWS, respectively. Compared with Fig. 14, we can see a clearer interference pattern in Fig. 15. Figures 16 and 17 show the calculated and measured frequency-amplitude characteristics in front of the center of the loudspeaker array, resepc-
Fig. 14: Calculated secondary wavefront (1600 Hz).

Fig. 15: Measured secondary wavefront (1600 Hz).

Fig. 16: Calculated frequency amplitude characteristics in front of center of loudspeaker array.

Fig. 17: Measured frequency amplitude characteristics in front of center of loudspeaker array.

Fig. 18: Measured frequency amplitude characteristics in front of center of loudspeaker array.

Fig. 19: Measured frequency amplitude characteristics in front of center of loudspeaker array.

Fig. 20: Measured frequency amplitude characteristics in front of center of loudspeaker array.

Fig. 21: Measured frequency amplitude characteristics in front of center of loudspeaker array.

Fig. 22: Measured frequency amplitude characteristics in front of center of loudspeaker array.

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Fig. 14: Calculated secondary wavefront (1600 Hz).

Fig. 15: Measured secondary wavefront (1600 Hz).

Fig. 16: Calculated frequency amplitude characteristics in front of center of loudspeaker array.

Fig. 17: Measured frequency amplitude characteristics in front of center of loudspeaker array.

Effect of room reflection

In the soundproof room used in this measurement, we consider that the wave reflected by the wall affects the secondary synthesized wavefront. The reflection surface nearest to the source is the floor of this room. The relation between the direct and reflected waves generated by the primary source is obtained by the image method [13], where the line \( z = 0 \) corresponds to the floor. In the image method the reflected wave is generated by an imaginary source located at the reflection of the primary source in the line \( y = 0 \). In the image method, we can regard the reflected wave as a direct wave generated from this imaginary point source. Using the image method, we can consider the effect of waves reflected from the floor, and we calculate the primary wavefront using this method. Figure 18 and Table 3 show the calculation conditions.

Figure 19 shows the frequency characteristics of the wavefront calculated with the primary source and the reflected wave obtained by the image method. This result shows that the wave reflected from the floor surface causes an interference pattern in the frequency characteristics, which tends to broaden as the observation point becomes more distant from the primary source. We consider that the secondary wavefront affects the wave reflected from the floor surface. Therefore, we remove the

\[
v(t) = \begin{cases} 
0.5 - 0.5 \cos 2\pi \frac{t}{T_D + T_{R1}} & \text{(if } 0 \leq t \leq T_D + T_{R1}) \\
0 & \text{(otherwise)}
\end{cases}
\]

where \( T_D \) is the time of arrival at the nearest observation point to the primary source, and \( T_{R1} \) is the arrival time interval of the first early reflection wavefront at the observation point. Figure 20 shows an impulse response as an example including the wave reflected from the floor surface and the definitions of \( T_D \) and \( T_{R1} \).

Figure 21 shows the frequency characteristics of the secondary wavefront in front of the primary source. According to Fig. 21, the wavefront reflected from the floor surface is clarified to be the secondary wavefront.

Figure 22 shows the secondary wavefront after the removal of the wavefront reflected in front of the primary sound source. By comparing this figure with Fig. 15, we can conclude that the reflected wavefront caused the interference observed in the measurement results because the wavefront interference is less evident in Fig. 22. In addition, DMCWS is performed above the WFS aliasing frequency (1417 Hz) in Fig. 22.
Fig. 18: Calculation conditions used to determine the effect of the wave reflected from the floor surface.

Table 3: Calculation conditions for primary source wavefront and the wavefront reflected from the floor surface

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room temperature</td>
<td>15°C</td>
</tr>
<tr>
<td>Sound velocity ( c )</td>
<td>340.64 m/s</td>
</tr>
<tr>
<td>Sampling frequency ( f_s )</td>
<td>48 kHz</td>
</tr>
<tr>
<td>Primary sound source model</td>
<td>Point source</td>
</tr>
</tbody>
</table>

4.6. Effect of loudspeaker frequency characteristics on measured wavefront

Figure 23 shows the frequency characteristics of the measured before and after the removal of the reflected wavefront and that of the loudspeaker (Soundevice SD-0.6) at the most nearest observation point of \((x_0, y_0) = (1.2, 0.3)\) m. According to the results, the measured characteristics are similar to those of the secondary source loudspeaker in the low-frequency subband with frequencies of up to 1600 Hz. Thus, the amplitude error of the measured wavefront can be attributed to the frequency-amplitude characteristics of the loudspeaker.

5. CONCLUSIONS

In this paper it has been shown that the accuracy of the synthesized secondary wavefront with comparing calculation results of DMCWS and WFS and the wavefront measurement results of DMCWS in an actual environment. Numerical wavefront calculations clarified that DMCWS has a larger listening area with fewer amplitude and phase errors than WFS. Our wavefront measurement system and our algorithm using impulse responses measured in an acoustically iso-

6. ACKNOWLEDGMENT

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7. REFERENCES

Fig. 21: Frequency-amplitude characteristics of the secondary wavefront in front of the primary sound source after the removal of the wavefront reflected from the floor surface.

Fig. 22: Secondary wavefront after removal of reflected wavefront (1600 Hz).


